33-755 QUANTUM I Fall 2020

Class meets MWF 9:20 AM in Doherty Hall 2105 and remote online, and F 5:20 PM remote online
Professor: Mike Widom, Office 6305 Wean Hall, e-mail: widom@cmu.edu
Office Hours: Questions and discussion on Canvas/Piazza. Zoom appointments available by e-mail request.

Note related to the coronavirus: This class will meet in person for the first week of the semester, then transition to fully remote. Zoom meeting links and recordings will be posted on Canvas. Class discussion (student questions, etc.) will take place on Piazza.

This course introduces quantum mechanics at a first-semester introductory graduate level. Prior familiarity with basic principles and applications of quantum mechanics will be assumed at an advanced undergraduate level. Special attention will be paid to fundamental postulates of quantum mechanics and their consequences, in addition to practical applications to physical phenomena. Specific applications include: two-level systems; quantum information; harmonic oscillators, phonons and blackbody radiation; charged particles in magnetic fields; hydrogen atom. A course web site at http://euler.phys.cmu.edu/widom/teaching/33-755 contains this syllabus plus links to day-by-day lecture coverage and weekly homework assignments.

Books: The principal content of the course will be drawn from two books:
1. Cohen-Tannoudji, Diu and Laloe, *Quantum Mechanics*, vol. 1.
2. Griffiths, *Consistent Quantum Theory*. Available online at http://quantum.phys.cmu.edu/CQT. Also see a short and simplified summary of the consistent histories approach.

Grading: Letter grades will be based on weekly homework assignments (20%), two midterms (20% each) and a final exam (40%). Students are welcome to collaborate on homework but the final solutions must be your own work, written up separately. Homework assignments are listed at http://euler.phys.cmu.edu/widom/teaching/33-755/hw.html. Your work will be handed in and grades will be listed on the course page on Canvas. Dates for midterm and final exams will be announced as they become available.

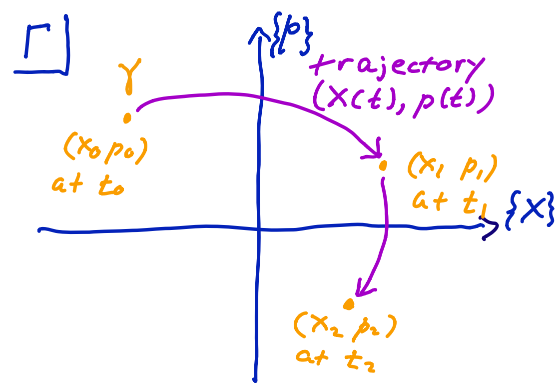
Course outline: We will cover the topics listed below in approximately the given time frames. Detailed class coverage can be found at http://euler.phys.cmu.edu/widom/teaching/33-755/coverage/html.

- Weeks 1-2. Hilbert space and physical properties. Composite systems
- Week 3. Unitary dynamics, spin 1/2 in magnetic field
- Weeks 4-5. Stochastic processes, sample spaces and interference
- Weeks 6-7. Multiple times, consistency, measurements, Quantum information
- Week 8. Wave mechanics, path integrals
- Weeks 9-11. Harmonic oscillator, phonons and blackbody
- Weeks 12-15. Angular momentum and hydrogen atom

Hilbert Space

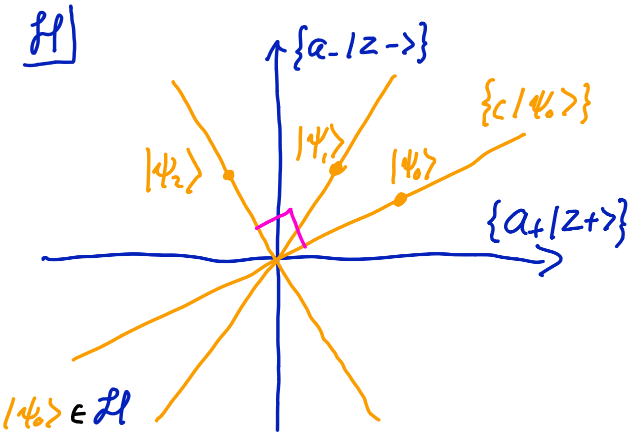
Classical Phase Space

- real axes
- states are points



Hilbert Space

- complex vector space
- states are rays
- different states: $|\psi_0\rangle, |\psi_1\rangle$
- orthogonal states: $\langle\psi_2|\psi_1\rangle = 0$
- closed under scalar multiplication $c|\psi_0\rangle \in \mathcal{H}$
- " " addition $|\psi_1\rangle + |\psi_2\rangle \in \mathcal{H}$



Adjoint $\dagger$: "ket" $|\psi\rangle \in \mathcal{H} \longrightarrow$ "bra" $\langle\psi| \equiv (|\psi\rangle)^\dagger \in \mathcal{H}^\dagger$

Inner Product $\langle\psi| : |\varphi\rangle \longrightarrow \langle\psi|\varphi\rangle \in \mathbb{C}$

$\langle\psi| : |\psi\rangle \longrightarrow \langle\psi|\psi\rangle \equiv \|\psi\|^2 \geq 0$ "norm"

e.g. $|\psi\rangle = a_+|z+\rangle + a_-|z-\rangle \qquad |\varphi\rangle = b_+|z+\rangle + b_-|z-\rangle$

$\langle\psi| = a_+^*\langle z+| + a_-^*\langle z-| \qquad \langle\psi|\psi\rangle = |a_+|^2 + |a_-|^2$

$\langle\psi|\varphi\rangle = a_+^* b_+ + a_-^* b_- = \langle\varphi|\psi\rangle^*$

Operators: $A: \mathcal{H} \to \mathcal{H}$

$A: |\psi\rangle \to |\varphi\rangle = A|\psi\rangle \equiv |A\psi\rangle$

"Matrix element" $\langle \chi | A | \psi \rangle \equiv \langle \chi | (A|\psi\rangle) = \langle \chi | \varphi \rangle$

Adjoint: $A^\dagger: \langle \chi | \to \langle \chi | A^\dagger \equiv (A|\chi\rangle)^\dagger$

e.g. $\langle \chi | A^\dagger | \psi \rangle = (\langle \chi | A^\dagger)|\psi\rangle = [\langle \psi | (A|\chi\rangle)]^* = [\langle \psi | A\chi \rangle]^*$

$\therefore \langle \chi | A^\dagger = \langle A\chi |$

Normal Operators: $N^\dagger N = N N^\dagger$ ($\underset{\text{Hermitian}}{H = H^\dagger}$, $\underset{\text{Unitary}}{UU^\dagger = U^\dagger U = I}$)

1. If $|\psi\rangle$ is eigenvector of N with eigenvalue λ $\quad (N|\psi\rangle = \lambda|\psi\rangle)$
Then " " $N^\dagger$ " " λ^* $\quad (N^\dagger|\psi\rangle = \lambda^*|\psi\rangle)$

proof: $\|(N-\lambda)|\psi\rangle\|^2 = 0 = \langle \psi | (N^\dagger - \lambda^*)(N-\lambda)|\psi\rangle$

$= \langle \psi | (N-\lambda)(N^\dagger - \lambda^*)|\psi\rangle = \|(N^\dagger - \lambda^*)|\psi\rangle\|^2$

2. If $N|\psi_i\rangle = \lambda_i|\psi_i\rangle$ and $N|\psi_j\rangle = \lambda_j|\psi_j\rangle$ with $\lambda_i \neq \lambda_j$ then $\langle \psi_i | \psi_j \rangle = 0$

proof: $\lambda_j \langle \psi_i | \psi_j \rangle = \langle \psi_i | (N|\psi_j\rangle) = (\langle \psi_i | N)|\psi_j\rangle$

$= (N^\dagger|\psi_i\rangle)^* |\psi_j\rangle = \lambda_i \langle \psi_i | \psi_j \rangle$

3. H has real eigenvalues

proof: $H|\psi\rangle = \lambda|\psi\rangle$, $\langle \psi | H^\dagger = \lambda^* \langle \psi |$, $\langle \psi | H | \psi \rangle = \langle \psi | H^\dagger | \psi \rangle \Rightarrow \lambda = \lambda^*$

4. Eigenvalues of U obey $|\lambda|^2 = 1$

proof: $\langle \psi | \psi \rangle = \langle \psi | I | \psi \rangle = \langle \psi | U^\dagger U | \psi \rangle = \lambda^* \lambda \langle \psi | \psi \rangle$

5. Eigenvectors span $\mathcal{H}$ (completeness)