EPR (1935) state $|\psi_0\rangle = \frac{1}{\sqrt{2}}(|+-\rangle - |-+\rangle)$ challenges "local realism" (claim that all possible properties exist and are locally determined)

Bell inequality (1964) Local realism demands correlation inequalities hold that quantum mechanics violate.

Aspect (1982) Confirmed violation of Bell inequality

Greenberger-Horn-Zeilinger "Experiment" (1989)

Non-probabilistic variant of Bell (See Mermin, Physics Today 1990)

Spin-1/2 : $|x\pm\rangle = \frac{1}{\sqrt{2}}(|z+\rangle \pm |z-\rangle)$ $\quad |y\pm\rangle = \frac{1}{\sqrt{2}}(|z+\rangle \pm i|z-\rangle)$

Take $\sigma_x = S_x/(\hbar/2)$ as scaled spin operator (Pauli matrix)

$\sigma_x|x+\rangle = |x+\rangle$, $\sigma_x|x-\rangle = -|x-\rangle$ same for σ_y and σ_z

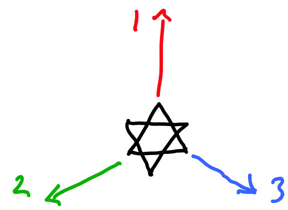
Note: $\sigma_x|z+\rangle = \sigma_x \cdot \frac{1}{\sqrt{2}}(|x+\rangle + |x-\rangle) = |z-\rangle$, $\quad \sigma_x|z-\rangle = |z+\rangle$

$\sigma_y|z+\rangle = \sigma_y \cdot \frac{1}{\sqrt{2}}(|y+\rangle + i|y-\rangle) = i|z-\rangle$, $\quad \sigma_y|z-\rangle = -i|z+\rangle$

and note: $\left.\begin{array}{l}\sigma_x\sigma_y|z\pm\rangle = \pm i|z\pm\rangle \\ \sigma_y\sigma_x|z\pm\rangle = \mp i|z\pm\rangle\end{array}\right\} \Rightarrow \sigma_x\sigma_y = -\sigma_y\sigma_x = i\sigma_z$

Three-particle decay:

$$|\psi\rangle = |z+z+z+\rangle - |z-z-z-\rangle$$



Define: $X_1 \equiv \sigma_x^1 \sigma_y^2 \sigma_y^3 \qquad X_2 \equiv \sigma_y^1 \sigma_x^2 \sigma_y^3 \qquad X_3 \equiv \sigma_y^1 \sigma_y^2 \sigma_x^3$

Note: $X_1 |z+z+z+\rangle = i^2 |z-z-z-\rangle, \quad X_1 |z-z-z-\rangle = (-i)^2 |z+z+z+\rangle$

$$\therefore\ X_1|\psi\rangle = {+1}\,|\psi\rangle \qquad X_2|\psi\rangle = |\psi\rangle \qquad X_3|\psi\rangle = |\psi\rangle$$

Let $m_x^1 = \pm 1$ be the value of S_x^1, $m_y^1 = \pm 1$, $m_x^2 = \pm 1$, etc.

Local realism says $m_x^1\ m_y^1\ m_x^2\ m_y^2\ m_x^3\ m_y^3$ all exist simultaneously even if we don't measure them.

We could measure S_x^1, S_y^2, S_y^3 to obtain $m_x^1 m_y^2 m_y^3 = \pm 1$

but $X_1|\psi\rangle = |\psi\rangle \implies m_x^1 m_y^2 m_y^3 = +1$ whenever measured

also $X_2|\psi\rangle = |\psi\rangle \implies m_y^1 m_x^2 m_y^3 = +1$ " "

and $X_3|\psi\rangle = |\psi\rangle \implies m_y^1 m_y^2 m_x^3 = +1$ " "

Then $+1 = (+1)^3 = (m_x^1 m_y^2 m_y^3)(m_y^1 m_x^2 m_y^3)(m_y^1 m_y^2 m_x^3)$

$$= (m_x^1 m_x^2 m_x^3)(m_y^1 m_y^2 m_y^3)^2 = m_x^1 m_x^2 m_x^3$$

i.e. $X_1 X_2 X_3 |\psi\rangle = m_x^1 m_x^2 m_x^3 |\psi\rangle = (+1)\cdot|\psi\rangle$

Now consider $\Sigma_{123} \equiv \sigma_x^1 \sigma_x^2 \sigma_x^3$ $\Sigma_{123}|\psi\rangle = m_x^1 m_x^2 m_x^3$

Compare $\Sigma_1 \Sigma_2 \Sigma_3 = (\sigma_x^1 \sigma_y^2 \sigma_y^3)(\sigma_y^1 \sigma_x^2 \sigma_y^3)(\sigma_y^1 \sigma_y^2 \sigma_x^3)$

$\sigma_y^2 \sigma_x^2 = -\sigma_x^2 \sigma_y^2$
$\sigma_y^3 \sigma_x^3 = -\sigma_x^3 \sigma_y^3$
(twice)

$$= (-1)^3 (\sigma_x^1 \sigma_x^2 \sigma_x^3)(\sigma_y^1 \sigma_y^2 \sigma_y^3)^2$$

$$= -\Sigma_{123}$$

So, $\Sigma_{123}|\psi\rangle = -\Sigma_1 \Sigma_2 \Sigma_3 |\psi\rangle = -|\psi\rangle \Rightarrow m_x^1 m_x^2 m_x^3 = -1$ ☆☆

☆☆ Contradicts ☆

☆ is invalid reasoning because m_x^1 and m_y^1 are not simultaneously defined ... local realism is not correct

i.e. $\left.\begin{array}{l} [x_1 +][y_1 +] \\ [x_1 +][y_1 -] \\ [x_1 -][y_1 +] \\ [x_1 -][y_1 -] \end{array}\right\}$ these are not properties