Spectral decomposition of T:

$H = \sum_j E_j P_j$ $\quad \{E_j\}$ discrete energy levels $\quad P_j \to E_j$ subspace

$$e^{\alpha H} = I + \alpha \sum_j E_j P_j + \frac{1}{2}\alpha^2 \underbrace{\sum_j E_j P_j \sum_k E_k P_k}_{\sum_j E_j^2 P_j \text{ because } P_j P_k = \delta_{jk} P_k} + \cdots$$

$$e^{\alpha H} = \sum_j e^{\alpha E_j} P_j$$

$$\therefore T(t-t') = e^{(-i/\hbar)(t-t')H} = \sum_j e^{(-i/\hbar)(t-t')E_j} P_j$$

Multiple time Born rule:

$$Pr(P_t \mid \psi_{t'}) = Pr(P_t \mid \psi_t) = \| P\, T(t,t') \mid \psi_{t'} \rangle \|^2$$

Example: $Pr(|\psi_t\rangle \mid |\psi_{t'}\rangle) = \| \langle \psi_t |\, T(t,t') \mid \psi_{t'} \rangle \|^2$

$= \| \langle \psi_t \mid \psi_t \rangle \|^2$

$= 1$