Spin-1/2 in magnetic field

$H = -\vec{\mu}\cdot\vec{B}$ gyromagnetic ratio

Magnetic moment $\vec{\mu} = \gamma\vec{S} = \frac{1}{2}\hbar\gamma\vec{\sigma}$

Magnetic field $\vec{B} = B\hat{n}$ (constant)

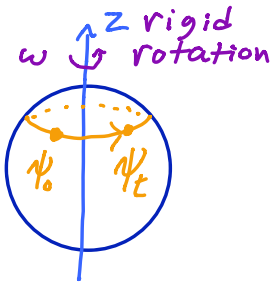
Larmor frequency $\omega = -\gamma B \Rightarrow H = \frac{1}{2}\hbar\omega\,\hat{n}\cdot\vec{\sigma}$

Let $\hat{n} = \hat{z} \Rightarrow H = \frac{1}{2}\hbar\omega\sigma_z = \frac{1}{2}\hbar\omega\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$

Time evolution operator $T(t) = e^{-itH/\hbar} = \begin{pmatrix}e^{-i\omega t/2} & 0\\ 0 & e^{i\omega t/2}\end{pmatrix}$

Let $|\psi_0\rangle = |\hat{u}+\rangle = \begin{pmatrix}1\\ \beta\end{pmatrix}$

$\Rightarrow |\psi_t\rangle = T(t)|\psi_0\rangle = e^{-i\omega t/2}\begin{pmatrix}1\\ \beta e^{i\omega t}\end{pmatrix}$ (ignore $e^{-i\omega t/2}$)



Arbitrary $\hat{n}$

$T(t) = e^{-i(\omega t/2)\hat{n}\cdot\vec{\sigma}} = \cos\left(\frac{\omega t}{2}\right)\mathbb{I} - i\sin\left(\frac{\omega t}{2}\right)\hat{n}\cdot\vec{\sigma}$

e.g. $\hat{n} = \hat{y} \Rightarrow T(t) = \begin{pmatrix}\cos\omega t/2 & -\sin\omega t/2\\ \sin\omega t/2 & \cos\omega t/2\end{pmatrix}$

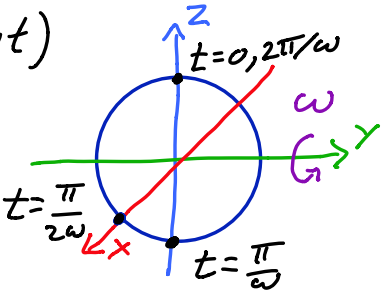
$|\psi_0\rangle = |z+\rangle \rightarrow |\psi_t\rangle = T(t)|\psi_0\rangle = \begin{pmatrix}\cos\omega t/2\\ \sin\omega t/2\end{pmatrix}$

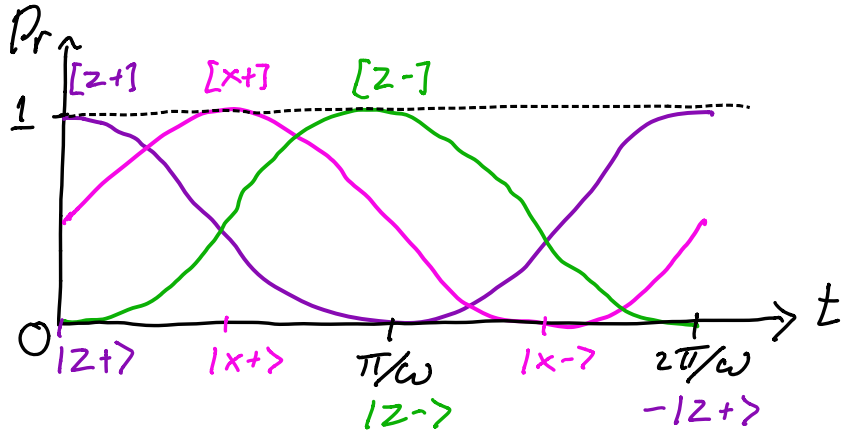
$P_r\left([z+]_t\,|\,\psi_0\right) = \langle\psi_0|T^\dagger[z+]T|\psi_0\rangle = \langle\psi_t|z+\rangle\langle z+|\psi_t\rangle$

$= \cos^2\omega t/2 = \frac{1}{2}(1+\cos\omega t)$

$P_r\left([z-]_t\,|\,\psi_0\right) = \sin^2\omega t/2$

$P_r\left([x+]_t\,|\,\psi_0\right) = \frac{1}{2} + \cos\frac{\omega t}{2}\sin\frac{\omega t}{2}$





Rabi Oscillations