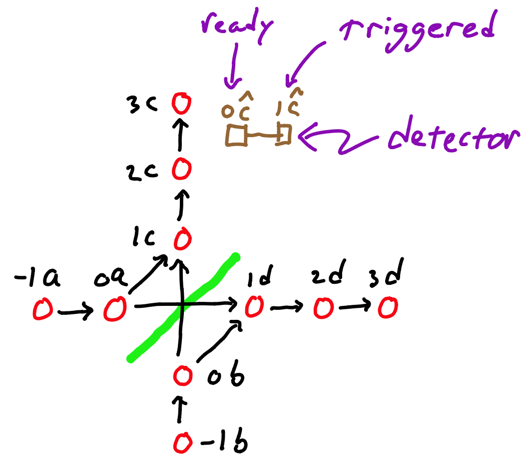
Measurement



Beam splitter with detector

$$\mathcal{H} = \mathcal{H}_p \otimes \mathcal{H}_d$$

Photon: basis $\{|mz\rangle\}$ (for $\mathcal{H}_p$)

detector: basis $\{|0\hat{c}\rangle, |1\hat{c}\rangle\}$ (for $\mathcal{H}_d$)

$$T = S\,R$$

$$R = I_p \otimes I_d \text{ except: } R|2c, n\hat{c}\rangle = |2c, (1-n)\hat{c}\rangle$$

$$S|mz, n\hat{c}\rangle = |(m+1)z, n\hat{c}\rangle$$

$$S|0a, n\hat{c}\rangle = \frac{1}{\sqrt{2}}\left(|1c, n\hat{c}\rangle + |1d, n\hat{c}\rangle\right)$$

Unitary evolution:

$$|\psi_0\rangle = |0a, 0\hat{c}\rangle$$

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\left(|1c, 0\hat{c}\rangle + |1d, 0\hat{c}\rangle\right)$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\left(|2c, 0\hat{c}\rangle + |2d, 0\hat{c}\rangle\right)$$

$$|\psi_3\rangle = \frac{1}{\sqrt{2}}\left(|3c, 1\hat{c}\rangle + |3d, 0\hat{c}\rangle\right)$$

Consistent history family:

Basis set $\{[mz, n\hat{c}]\}$ at each time

Histories with nonzero chain kets

$$Y^c = [\psi_0] \odot [1c, 0\hat{c}] \odot [2c, 0\hat{c}] \odot [3c, 1\hat{c}] \qquad |Y^c\rangle = \frac{1}{\sqrt{2}}|3c, 1\hat{c}\rangle \qquad Pr = 1/2$$

$$Y^d = [\psi_0] \odot [1d, 0\hat{c}] \odot [2d, 0\hat{c}] \odot [3d, 0\hat{c}] \qquad |Y^d\rangle = \frac{1}{\sqrt{2}}|3d, 0\hat{c}\rangle \qquad Pr = 1/2$$

Others vanish, e.g. $Pr([\psi_0] \odot \cdots \odot [3d, 1\hat{c}]) = 0$

$$Pr([1\hat{c}]_3 \mid [2c]_2) = Pr(Y^c) \div Pr([2c]_2) = \tfrac{1}{2} \div \tfrac{1}{2} = 1$$

$$Pr([2c]_2 \mid [1\hat{c}]_3) = Pr(Y^c) \div Pr([1\hat{c}]_3) = \tfrac{1}{2} \div \tfrac{1}{2} = 1$$

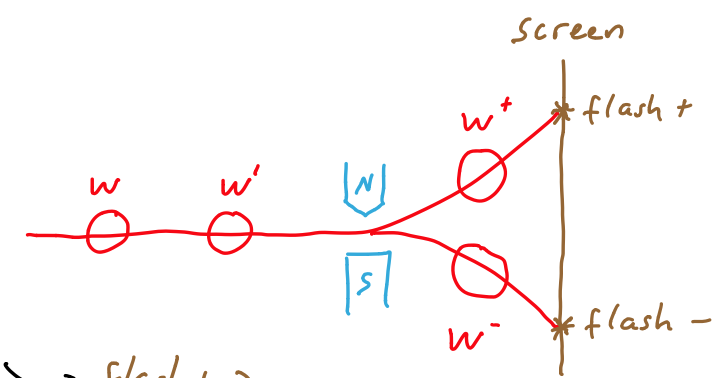
Unitary history family: $[\psi_0]\odot[\psi_1]\odot[\psi_2]\cdots$

$$[\psi_0]\odot(I_1-[\psi_1])\odot[\psi_2]\cdots$$
$$\vdots$$

Fine family but:
$$\left.\begin{aligned}[\psi_t][tz,\hat{nc}] &= \tfrac{1}{\sqrt{2}}|\psi_t\rangle\langle tz,\hat{nc}| \\ [tz,\hat{nc}][\psi_t] &= \tfrac{1}{\sqrt{2}}|tz,\hat{nc}\rangle\langle\psi_t|\end{aligned}\right\}\neq \text{ for } t\geq 1$$

Measurement of spin-1/2

Stern-Gerlach experiment:



Unitary evolution:

$$\left.\begin{aligned}|z+,w\rangle \to |z+,w'\rangle \to |z+,w^+\rangle &\to \text{flash} + \\ |z-,w\rangle \to |z-,w'\rangle \to |z-,w^-\rangle &\to \text{flash} -\end{aligned}\right\}\text{ destructive}$$

nondestructive

What if $|\psi_0\rangle = |x+,w\rangle = \frac{1}{\sqrt{2}}\left(|z+,w\rangle + |z-,w\rangle\right)$?

$$\underbrace{|x+,w\rangle}_{|\psi_0\rangle} \to \underbrace{|x+,w'\rangle}_{|\psi_1\rangle} \longrightarrow \underbrace{\tfrac{1}{\sqrt{2}}\left(|z+,w^+\rangle + |z-,w^-\rangle\right)}_{|\psi_2\rangle}$$

Unitary history $[\psi_0]\odot[\psi_1]\odot[\psi_2]$

Measurement outcomes $\{I_s\otimes[w^+],\ I_s\otimes[w^-]\}$

incompatible because $[\psi_2][w^\pm]\neq[w^\pm][\psi_2]$

Alternate family $[\psi_0]\odot[\psi_1]\odot\begin{cases}[z+,w^+]\\ [z-,w^-]\end{cases}$

Last try $[\psi_0]\odot\begin{cases}[z+,w']\odot[z+,w^+] \leftarrow Y^+\\ [z-,w']\odot[z-,w^-] \leftarrow Y^-\end{cases}$

$$Pr([Z^+]_1 \mid [W^+]_2) = Pr([Z^+]_1, [W^+]_2) \div Pr([W^+]_2) = Pr(Y^+) \div Pr(Y^+) = 1 \checkmark$$

$\therefore$ true measurement