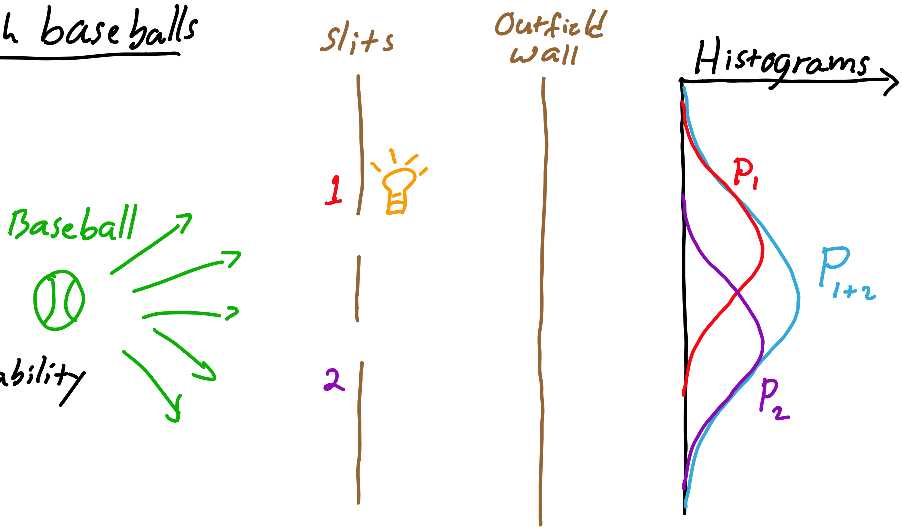
Experiments with baseballs

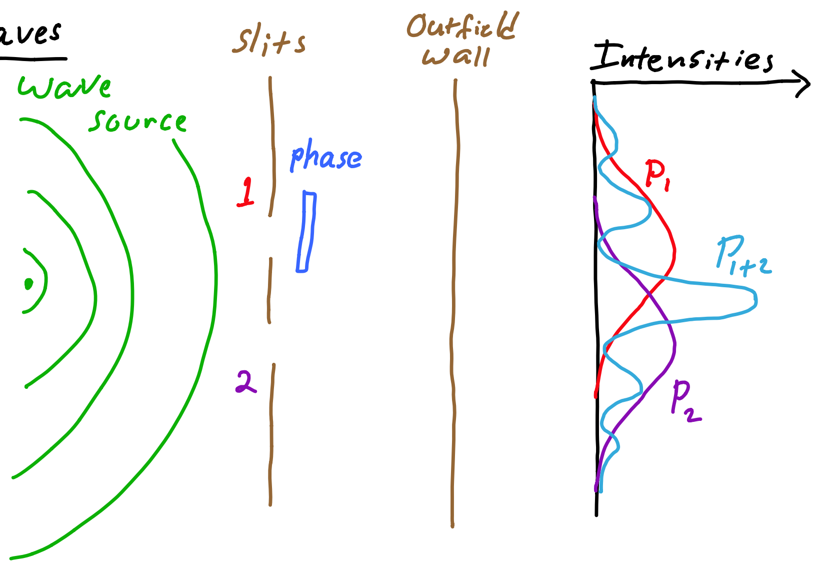
1. Discrete events
2. $P_{1+2} = P_1 + P_2$
3. Can detect slit passage with light
4. Conditional probability

   $P_{1+2} \mid \text{slit } 1 = P_1$



Experiments with waves

1. Continuous events
2. $P_{1+2} \neq P_1 + P_2$
3. Cannot define slit 1 vs. slit 2
4. Closing slit destroys interference pattern
5. phase shifter shifts interference pattern



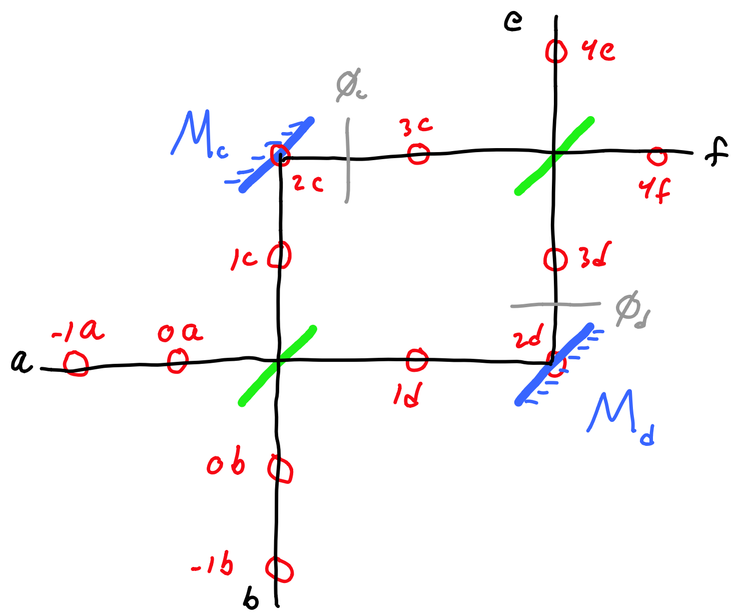
Experiments with electrons

1. Individual events
2. $P_{1+2} \neq P_1 + P_2$
3. Can detect slit, but when detected $P_{1+2} = P_1 + P_2$!
4. Detect passage but not slit: $P_{1+2} \neq P_1 + P_2$
5. Weak detection $\Longrightarrow$ partial interference

Q1: Electron passes slit 1 or 2 but not both?

Q2: Electron goes through both slits?

Simplified geometry: Mach-Zehnder interferometer



$T = S$, $S|mz\rangle = |(m+1)z\rangle$

except:

$S|0a\rangle = \frac{1}{\sqrt{2}}(|1c\rangle + |1d\rangle)$

$S|0b\rangle = \frac{1}{\sqrt{2}}(-|1c\rangle + |1d\rangle)$

$S|3c\rangle = \frac{1}{\sqrt{2}}(|4e\rangle + |4f\rangle)$

$S|3d\rangle = \frac{1}{\sqrt{2}}(-|4e\rangle + |4f\rangle)$

Useful definitions:

$|m\bar{a}\rangle \equiv \frac{1}{\sqrt{2}}(|mc\rangle + |md\rangle)$

$|m\bar{b}\rangle \equiv \frac{1}{\sqrt{2}}(-|mc\rangle + |md\rangle)$

Unitary time evolution: $|0a\rangle \to |1\bar{a}\rangle \to |2\bar{a}\rangle \to |3\bar{a}\rangle \to |4f\rangle$

never $|4e\rangle$!

Phase shifter: $S|2c\rangle = e^{i\phi_c}|3c\rangle$, $S|2d\rangle = e^{i\phi_d}|3d\rangle$

Unitary evolution:

$$|0a\rangle \to |1\bar{a}\rangle \to |2\bar{a}\rangle \to \frac{1}{\sqrt{2}}(e^{i\phi_c}|3c\rangle + e^{i\phi_d}|3d\rangle)$$

$$\to \frac{1}{2}\left\{e^{i\phi_c}(|4e\rangle + |4f\rangle) + e^{i\phi_d}(-|4e\rangle + |4f\rangle)\right\}$$

Consider histories: $Y^e = [0a]_0 \odot [4e]_4$ $\quad Y^f = [0a]_0 \odot [4f]_4$

Chain kets: $|Y^e\rangle = [4e]T^4|0a\rangle = \frac{1}{2}(e^{i\phi_c} - e^{i\phi_d})|4e\rangle$ $\qquad \Delta \equiv \phi_c - \phi_d$

$|Y^f\rangle = [4f]T^4|0a\rangle = \frac{1}{2}(e^{i\phi_c} + e^{i\phi_d})|4f\rangle$

$$Pr(Y^e) = \langle Y^e|Y^e\rangle = \frac{1}{4}|e^{i\phi_c} - e^{i\phi_d}|^2 = \frac{1}{4}|e^{i\Delta/2}|^2|e^{i\Delta/2} - e^{-i\Delta/2}|^2 = \sin^2(\Delta/2)$$

$Pr(\gamma^f)$ $= \cos^2(\Delta/2)$