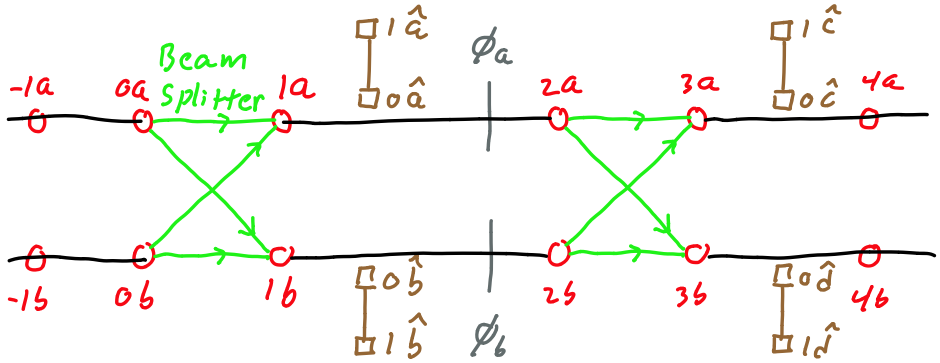
Mach-Zehnder with detectors



Unitary evolution (no detectors)

$$|\psi_0\rangle = |0a\rangle$$

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\left(|1a\rangle + |1b\rangle\right)$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\left(e^{i\phi_a}|2a\rangle + e^{i\phi_b}|2b\rangle\right)$$

$$|\psi_3\rangle = \frac{1}{2}\left(e^{i\phi_a} - e^{i\phi_b}\right)|3a\rangle + \frac{1}{2}\left(e^{i\phi_a} + e^{i\phi_b}\right)|3b\rangle$$

$$|\psi_4\rangle = \frac{1}{2}\left(e^{i\phi_a} - e^{i\phi_b}\right)|4a\rangle + \frac{1}{2}\left(e^{i\phi_a} + e^{i\phi_b}\right)|4b\rangle$$

$\phi_a - \phi_b$

Case 1 $\quad Pr([3a]_3) = \sin^2(\Delta/2)$ $\quad$ interference!

Case 2 $\quad$ Detector $\hat{a}$ $\quad T|1a, p\hat{a}\rangle = |2a, (1-p)\hat{a}\rangle$

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}\left(e^{i\phi_a}|2a, 1\hat{a}\rangle + e^{i\phi_b}|2b, 0\hat{a}\rangle\right)$$

$$|\Psi_3\rangle = \frac{1}{2}e^{i\phi_a}\left(|3a, 1\hat{a}\rangle + |3b, 1\hat{a}\rangle\right) + \frac{1}{2}e^{i\phi_b}\left(-|3a, 0\hat{a}\rangle + |3b, 0\hat{a}\rangle\right)$$

$$Pr([3a]_3) = \langle\Psi_3|\left([3a]\otimes I_{\hat{a}}\right)|\Psi_3\rangle = \frac{1}{2}\langle\Psi_3|\left(e^{i\phi_a}|3a, 1\hat{a}\rangle - e^{i\phi_b}|3a, 0\hat{a}\rangle\right)$$

$$= \frac{1}{4}\left\{|e^{i\phi_a}|^2 + |-e^{i\phi_b}|^2\right\} = \frac{1}{2}$$

No interference!

$\langle 1\hat{a}|0\hat{a}\rangle = 0$

Case 3 Detector $\hat{a}$ cannot distinguish a from b

$$T|1a, p\hat{a}\rangle = |2a, (1-p)\hat{a}\rangle \qquad T|1b, p\hat{a}\rangle = |2b, (1-p)\hat{a}\rangle$$

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}\left(e^{i\phi_a}|2a, 1\hat{a}\rangle + e^{i\phi_b}|2b, 1\hat{a}\rangle\right)$$

$Pr([3a]_3) = \sin^2(\Delta/2)$ interference reappears $\langle 1\hat{a}|1\hat{a}\rangle \neq 0$

Case 4 Two detectors $\hat{a}$ and $\hat{b}$

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}\left(e^{i\phi_a}|2a, 1\hat{a}\rangle + e^{i\phi_b}|2b, 1\hat{b}\rangle\right)$$ sloppy notation!

$Pr([3a]_3) = \frac{1}{2}$ No interference - a arm vs. b arm determined

Case 5 Two detectors $\hat{c}$ and $\hat{d}$

$$T|3a, p\hat{c}, q\hat{d}\rangle = |4a, (1-p)\hat{c}, q\hat{d}\rangle \quad T|3b, p\hat{c}, q\hat{d}\rangle = |4b, p\hat{c}, (1-q)\hat{d}\rangle$$

Consider histories $[\Psi_0] \odot \begin{Bmatrix}[1a] \\ [1b]\end{Bmatrix} \odot I_2 \odot I_3 \odot \begin{Bmatrix}[0\hat{c}] \\ [1\hat{c}]\end{Bmatrix}$

$\{Y^{mn},\ m = a, b,\ n = 0, 1\}$ ... but $\langle Y^{mn}|Y^{m'n'}\rangle \neq 0$

Replace $\odot \begin{Bmatrix}[0\hat{c}] \\ [1\hat{c}]\end{Bmatrix}$ with $\odot \begin{Bmatrix}[X^+] \\ [X^-]\end{Bmatrix}$

where $|X^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|1\hat{c}\, 0\hat{d}\rangle \pm |0\hat{c}\, 1\hat{d}\rangle\right)$