Quantum Information

Classical bit: 2 states $\{0,1\} \Rightarrow \log_2 2 = 1$ bit of information

Entropy due to lack of knowledge:

$p_0, p_1 \quad p_0 + p_1 = 1 \Rightarrow H(\{p\}) = \sum_j (-p_j \log_2 p_j)$ bits of entropy (Shannon entropy)

e.g. $p_0 = p_1 = \frac{1}{2}$, $H = -2 \cdot \frac{1}{2} \log_2 \frac{1}{2} = 1$

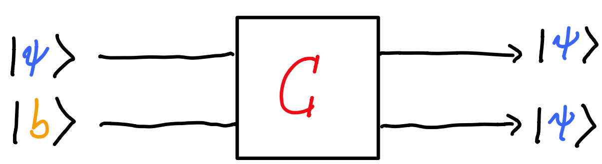
$\therefore$ Net information $1 - 1 = 0$ bits

Quantum bit: Density operator ρ: $S = -\mathrm{Tr}\, \rho \log_2 \rho$ ← von Neumann entropy

Pure state $|\psi\rangle \Rightarrow \rho$ is a 1-D projector $\lambda = 1, 0, 0, \cdots \Rightarrow S = 0$

$\therefore$ Net info $1 - 0 = 1$ qubit $\quad$ N qubits $= 2^N - 1$ bits of info
"dense coding"

No cloning theorem $\quad$ wish to copy $|\psi\rangle$ via unitary G



1. Given $|\psi\rangle$ and $|b\rangle$, G = rotation $|b\rangle \to |\psi\rangle$ always exists
2. Universal G to clone all $|\psi\rangle$?

Try: $|\chi\rangle \equiv G|\psi\rangle|b\rangle = |\psi\rangle|\psi\rangle \qquad |\chi'\rangle \equiv G|\psi'\rangle|b\rangle = |\psi'\rangle|\psi'\rangle$

$\langle\chi|\chi'\rangle = \langle\psi|\langle b|G^\dagger G|\psi'\rangle|b\rangle = \langle\psi|\psi'\rangle$

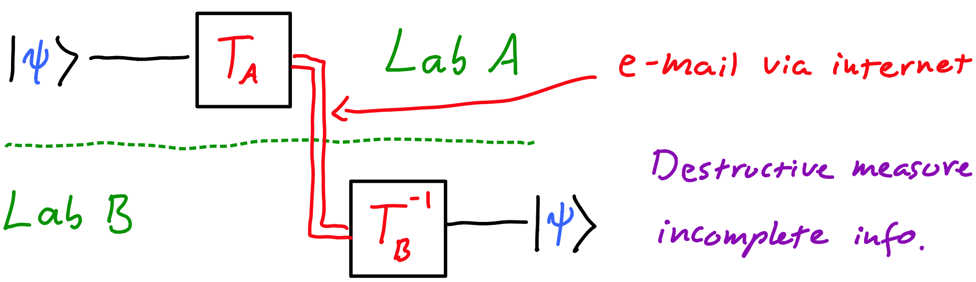
$= \langle\psi|\langle\psi|I|\psi'\rangle|\psi'\rangle = |\langle\psi|\psi'\rangle|^2$

$\Big\}$ $|\psi'\rangle \,/\!/\, |\psi\rangle$ or $|\psi'\rangle \perp |\psi\rangle$

$\therefore$ Cannot copy all of $\mathcal{H}$

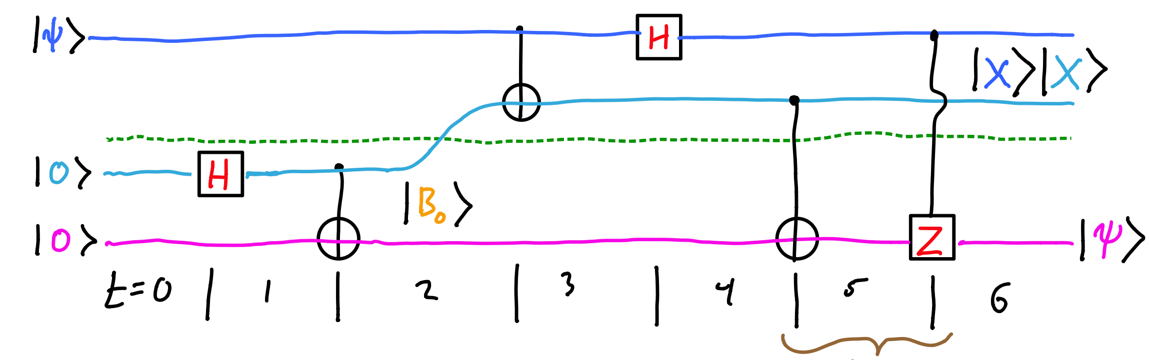
Teleportation

Transmit quantum state via classical channel



Destructive measurement T_A yields incomplete info.

Consider:



can replace quantum control with measurement followed by classical control

$$|\Psi_0\rangle = |\psi\rangle|0\rangle|0\rangle$$

$$|\Psi_1\rangle = |\psi\rangle \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle$$

$$|\Psi_2\rangle = |\psi\rangle|B_0\rangle \longleftarrow |B_0\rangle \equiv \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \text{ "Bell state"}$$

Let $|\psi\rangle = |0\rangle$ then treat $|1\rangle$ later: general $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$$|\Psi_2\rangle = |0\rangle|B_0\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |011\rangle)$$

$$|\Psi_3\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |011\rangle)$$

$$|\Psi_4\rangle = \frac{1}{2}(|000\rangle + |100\rangle + |011\rangle + |111\rangle)$$

$$|\Psi_5\rangle = \frac{1}{2}(|000\rangle + |100\rangle + |010\rangle + |110\rangle) = |x\rangle|x\rangle|0\rangle$$

$$|\Psi_6\rangle = |x\rangle|x\rangle|0\rangle = |x\rangle|x\rangle|\psi\rangle$$