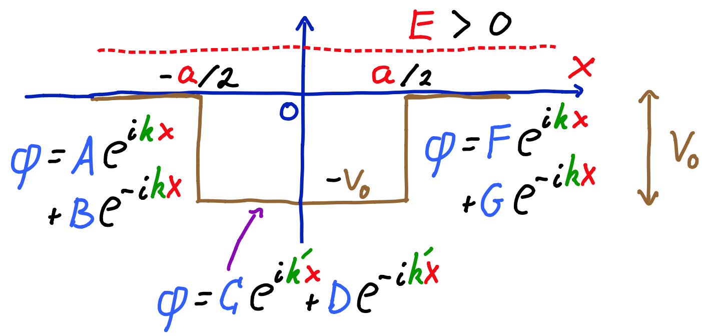
Square well scattering

Set $\hbar^2/2m = 1$

$k = \sqrt{E} \qquad k' = \sqrt{V_0 + E}$



Boundary Condition, $x = -a/2$:

φ continuous $\Rightarrow \quad A e^{-ika/2} + B e^{ika/2} = C e^{-ik'a/2} + D e^{ik'a/2}$

φ' continuous $\Rightarrow \quad kA e^{-ika/2} - kB e^{ika/2} = k'C e^{-ik'a/2} - k'D e^{ik'a/2}$

$\Rightarrow$ Linear relation $\quad R_k(-a/2)\begin{pmatrix} A \\ B \end{pmatrix} = R_{k'}(-a/2)\begin{pmatrix} C \\ D \end{pmatrix}$

$$R_k(x) = \begin{pmatrix} e^{ikx} & e^{-ikx} \\ ke^{ikx} & -ke^{-ikx} \end{pmatrix}$$

Boundary Condition, $x = a/2$: $\quad R_{k'}(a/2)\begin{pmatrix} C \\ D \end{pmatrix} = R_k(a/2)\begin{pmatrix} F \\ G \end{pmatrix}$

$\therefore \begin{pmatrix} A \\ B \end{pmatrix} = M\begin{pmatrix} F \\ G \end{pmatrix}, \quad M = R_k^{-1}(-a/2) R_{k'}(-a/2) R_{k'}^{-1}(a/2) R_k(a/2)$

Time indep. S.E. $\quad \left\{\left(-\frac{\partial^2}{\partial x^2} + (V(x) - E)\right)\varphi(x) = 0\right\}^*$

If φ solves T.I.S.E. then so does φ^* (time reversal invariance)

$(A e^{ikx})^* = A^* e^{-ikx} \leftarrow A^*$ plays role of B, etc.

$\begin{pmatrix} B^* \\ A^* \end{pmatrix} = M\begin{pmatrix} G^* \\ F^* \end{pmatrix} \Rightarrow \begin{pmatrix} B \\ A \end{pmatrix} = M^*\begin{pmatrix} G \\ F \end{pmatrix} \Rightarrow M = \begin{pmatrix} \gamma & \delta \\ \delta^* & \gamma^* \end{pmatrix}$ indep. of $V(x)$!

<u>Current Conservation</u>: $k\left(|A|^2-|B|^2\right)=k\left(|F|^2-|G|^2\right)$

$$=k\left(|\gamma|^2-|\delta|^2\right)\left(|F|^2-|G|^2\right)$$

$\therefore \det M=|\gamma|^2-|\delta|^2=1$

<u>Back to square well</u> $M_{11}=\gamma=e^{ika}\left[\cos(k'a)-i\,\frac{k^2+k'^2}{2kk'}\sin(k'a)\right]$

<u>Transmission</u> Set $A=1$ $G=0$ $\begin{pmatrix}1\\B\end{pmatrix}=M\begin{pmatrix}F\\0\end{pmatrix}$

$F=1/M_{11}=1/\gamma$ $T=|F|^2=\frac{1}{|\gamma|^2}$

$T(k)$ exhibits oscillations vs. k' with period $\frac{\pi}{a}$