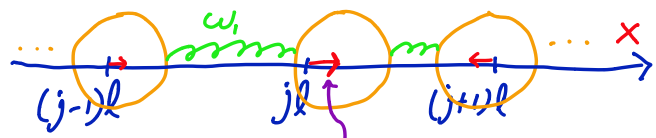
Continuum Limit



$$x_j(t) \longrightarrow u(x,t)$$

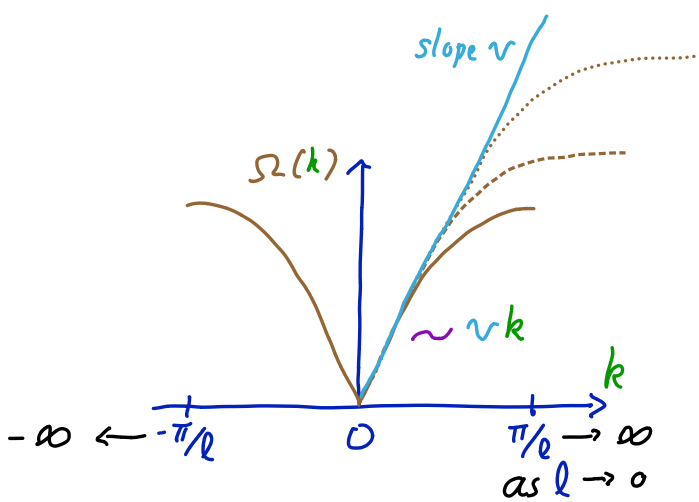
$$H = \sum_j \left\{ \tfrac{1}{2} m \dot{x}_j^2 + \tfrac{1}{2} m \omega_1^2 (x_{j+1} - x_j)^2 \right\}$$

$\underline{\ell \to 0:}$ $\quad \sum_j \longrightarrow \int \frac{dx}{\ell} \qquad x_{j+1} - x_j \longrightarrow \ell \frac{\partial u}{\partial x} \qquad \dot{x}_j \longrightarrow \frac{\partial u}{\partial t}$

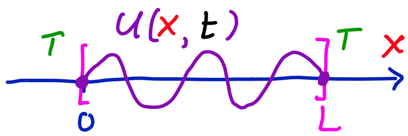
$$\mu = m/\ell \qquad K = m\omega_1^2 \ell$$

$$H \longrightarrow \int dx \left\{ \tfrac{1}{2}\mu \left(\frac{\partial u}{\partial t}\right)^2 + \tfrac{1}{2} K \left(\frac{\partial u}{\partial x}\right)^2 \right\}$$

$$\Omega(k) = 2\omega_1 \,|\sin(k\ell/2)| \longrightarrow (\omega_1 \ell) k = \sqrt{K/\mu}\, k \equiv v k$$



Blackbody Radiation



E+M field in equilibrium with walls of Container ... size L temperature T

$$u(x,t) \sim e^{\pm i(kx - \omega t)}$$

$$k = \frac{\pi}{L} n \qquad \omega = c\frac{\pi}{L} n$$

Density of States

Let $\mathcal{N}(\Omega)$ = # modes with $0 \leq \omega \leq \Omega$

$D(\omega)\,d\omega$ = # modes with $\omega \leq \omega' \leq \omega + d\omega$

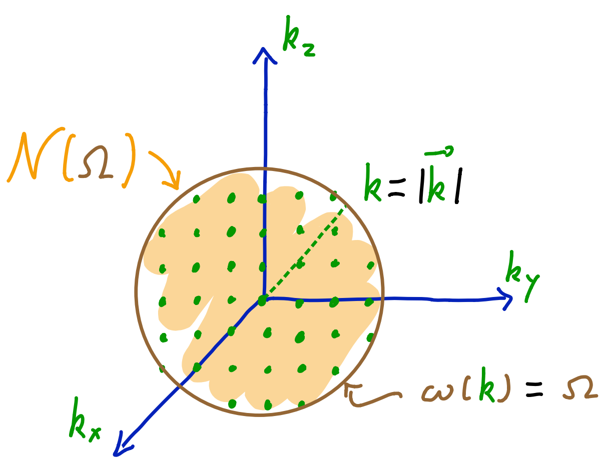
$\therefore\ D(\omega) = d\mathcal{N}/d\omega$

1-D: $\omega = c\frac{\pi}{L} n \qquad \mathcal{N}(\Omega) = L\Omega/\pi c \qquad D(\omega) = \frac{L}{\pi c}$ = constant

x2 for polarizations

includes 2x

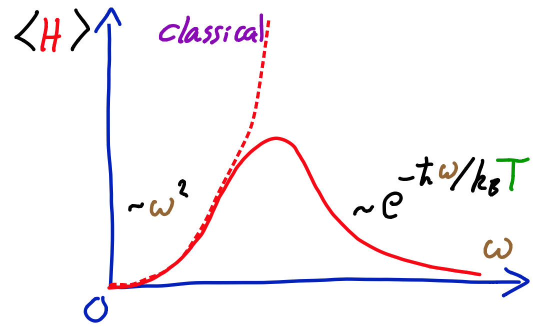
3-D: $\mathcal{N}(\Omega) = \frac{4\pi}{3}\left(\frac{\Omega L}{2\pi c}\right)^3 \qquad D(\omega) = \frac{8\pi}{8\pi^3} \cdot \frac{\omega^2 L^3}{c^3} = \frac{\omega^2}{\pi^2}\frac{1}{c^3}V$



Energy Density (per frequency interval, neglect zero point energy)

$$U(\omega) = D(\omega) \cdot \hbar\omega \cdot \langle N \rangle (\omega)$$

$$= \frac{\omega^2}{\pi^2} \frac{V}{c^3} \frac{\hbar\omega}{e^{\hbar\omega/k_B T} - 1}$$



classical $\quad U(\omega) = \frac{\omega^2}{\pi^2} \frac{V}{c^3} \frac{\hbar\omega}{\hbar\omega / k_B T}$

$$= \frac{\omega^2 k_B T V}{\pi^2 c^3}$$