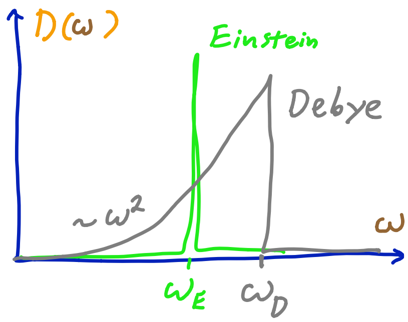
Heat capacity of solids

$U(\omega) = D(\omega)\cdot\hbar\omega\cdot\langle N\rangle(\omega)$

$U(T) = \int d\omega\ U(\omega)$



Einstein model: $D(\omega) = 3N\,\delta(\omega-\omega_E)$

$$C = \partial U/\partial T = 3N k_B \frac{(\hbar\omega_E/k_BT)^2 e^{\hbar\omega_E/k_BT}}{\left(e^{\hbar\omega_E/k_BT}-1\right)^2}$$

$$\longrightarrow \begin{cases} 3Nk_B & (k_BT \gg \hbar\omega_E) \\ \sim e^{-\hbar\omega_E/k_BT} & (k_BT \ll \hbar\omega_E) \end{cases}$$

Debye Model: $D(\omega) = \frac{3}{2}\frac{\omega^2}{\pi^2}\frac{1}{c^3}V$ up to ω_D

$$C \longrightarrow \begin{cases} 3Nk_B & (k_BT \gg \hbar\omega_E) \\ \sim T^3 & (k_BT \ll \hbar\omega_E) \end{cases}$$

Low T: excited modes: $\hbar\omega < k_BT$

$U \sim N(\omega = k_BT/\hbar)\cdot\hbar\omega \sim T^4$

$C \sim T^3$