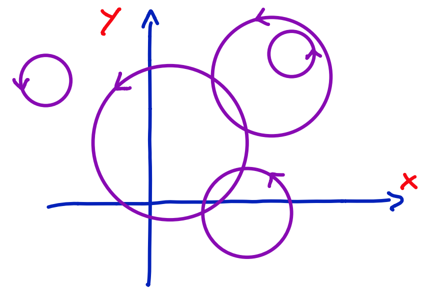
Charged particle in magnetic field $\vec{B} = B\hat{e}_z = \vec{\nabla} \times \vec{A}$

Classical: $\vec{f} = q\,\vec{v} \times \vec{B}$



Circle radius $\sim \sqrt{E}$

"Cyclotron"
frequency $\omega_c = -qB/m$

Hamiltonian $H = \frac{1}{2m}\left[\vec{p} - q\vec{A}\right]^2$ why? $\begin{cases} \dot{\vec{r}} = \frac{\partial H}{\partial \vec{p}} \\ \dot{\vec{p}} = -\frac{\partial H}{\partial \vec{r}} = q\,\dot{\vec{r}} \times \vec{B} \end{cases}$

Gauge Invariance add $\vec{\nabla}\chi(\vec{r})$ to $\vec{A}$ $\Rightarrow$ $\vec{B}$ unchanged

But: $H \rightarrow H' = \frac{1}{2m}\left[\vec{p} - q\vec{A} - q\vec{\nabla}\chi\right]^2$

same E, equivalent ψ

Quantum case? $H|\psi\rangle = E|\psi\rangle$ need $H'|\psi'\rangle = E|\psi'\rangle$

$\Rightarrow |\psi'\rangle = e^{iq\chi/\hbar}|\psi\rangle$

Note: transformation indep. of E $\Rightarrow$ holds for <u>all</u> quantum states

Coulomb gauge $\vec{\nabla}\cdot\vec{A} = 0$ $\vec{A} = -\frac{1}{2}\vec{r} \times \vec{B} = \frac{1}{2}B(x\hat{e}_y - y\hat{e}_x)$

$$H = \frac{P_x^2 + P_y^2}{2m} \underbrace{- \frac{qB}{2m}(XP_y - YP_x)}_{\frac{\omega_c}{2}L_z} + \underbrace{\frac{q^2B^2}{8m}(X^2 + Y^2)}_{\frac{1}{2}m\left(\frac{\omega_c}{2}\right)^2(X^2+Y^2)}$$

$$H = \underbrace{(N_d + N_g + 1)\hbar\left(\frac{\omega_c}{2}\right)}_{\text{2D H.O.}} + \underbrace{(N_d - N_g)\hbar\left(\frac{\omega_c}{2}\right)}_{L_z} = \left(N_d + \frac{1}{2}\right)\hbar\omega_c$$

States "Landau Levels"

$$|\psi_{n_d n_g}\rangle = \frac{1}{\sqrt{n_d!\,n_g!}}\,(a_d^+)^{n_d}(a_g^+)^{n_g}\,|\psi_{00}\rangle \qquad E_{n_d n_g} = (n_d + \tfrac{1}{2})\hbar\omega_c$$

a_d^+: control energy, counter-clockwise

a_g^+: control position, clockwise

degeneracy in n_g

$$|\psi_{00}\rangle = \frac{\kappa}{\sqrt{\pi}}\, e^{-\kappa^2(x^2+y^2)/2} \qquad \kappa^2 = qB/2\hbar$$

Alternate Solution in Landau gauge $\vec{A} = \vec{B} \times \hat{y}$

Breaks rotational symmetry

$$H = \frac{1}{2m}\left\{P_x^2 + (P_y - qBx)^2\right\}$$

translational symmetry in y not in x

$\Rightarrow$ eigenstates $\psi(x,y) = e^{iky}\varphi(x)$

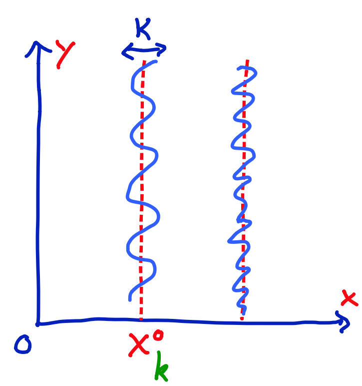
$$H\psi = e^{iky}\frac{1}{2m}\left\{p_x^2 + (\hbar k - qBx)^2\right\}\varphi(x) = E\,e^{iky}\varphi(x)$$

$\therefore\ \varphi(x)$ is eigenstate of $H_x = \frac{1}{2m}\left\{p_x^2 + \frac{1}{2}m\omega_c^2\left(x - \frac{\hbar k}{m\omega_c}\right)^2\right\}$

H.O. centered at x_k^0 $\qquad \frac{\hbar k}{m\omega_c} = x_k^0$

State $\psi_{nk}(x,y) = \frac{1}{\sqrt{n!}}(a^+)^n|\varphi_0\rangle e^{iky}$

Energy $E_n = (n + \frac{1}{2})\hbar\omega_c$ independent of k



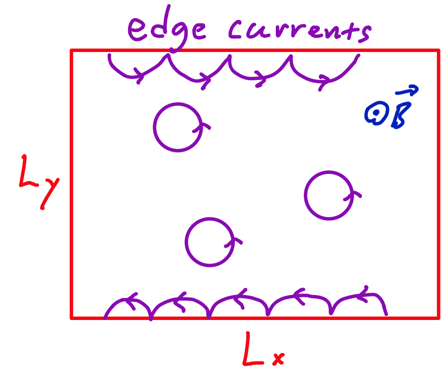
Degeneracy

$L_x \times L_y$ box with P.B.C. $\Rightarrow k = \frac{2\pi}{L_y} N$

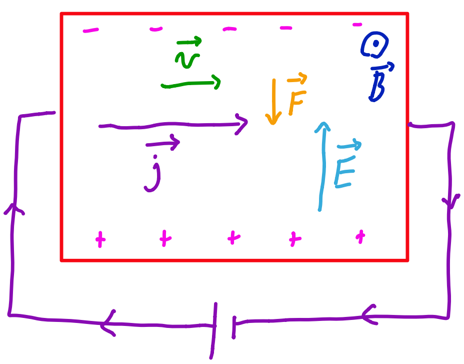
Box: $0 \le X^{\circ} < L_x \Rightarrow 0 \le N < \frac{m\omega_c L_x L_y}{2\pi\hbar}$

$\therefore$ degeneracy $D = \frac{qBA}{h} \equiv \Phi/\Phi_0$ $\quad \Phi = BA \quad \Phi_0 = h/q$ "flux quantum"

1 Landau level / flux quantum



Integer quantum Hall effect



density

Classical Hall resistivity $\rho_{yx} = E_y/j_x = B/qn$

Density of States