NAME: ____________________________________

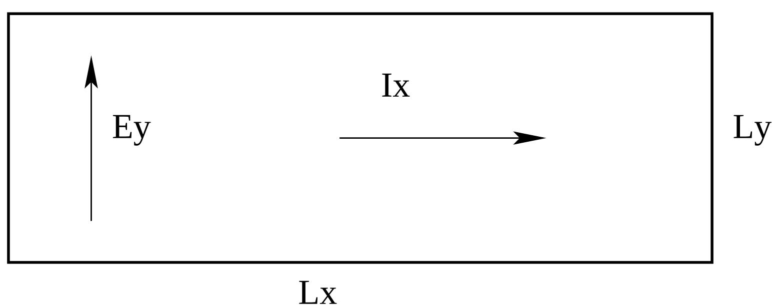
33-783 Solid State Physics   Midterm #1   Wednesday, October 4, 2017

1. Magnetoconductivity in the Hall geometry

Consider the Hall effect in a two dimensional channel of dimensions $L_x \times L_y$ (see figure below). The static magnetoconductivity tensor σ is defined so that

$$\begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

with $\sigma_{yx} = -\sigma_{xy}$. Recall that for Hall effect experiments in steady state, no current flows in the y-direction.



(a) Define the effective conductivity $\sigma_{eff} \equiv j_x/E_x$ and express σ_{eff} in terms of the elements of the magnetoconductivity tensor σ.

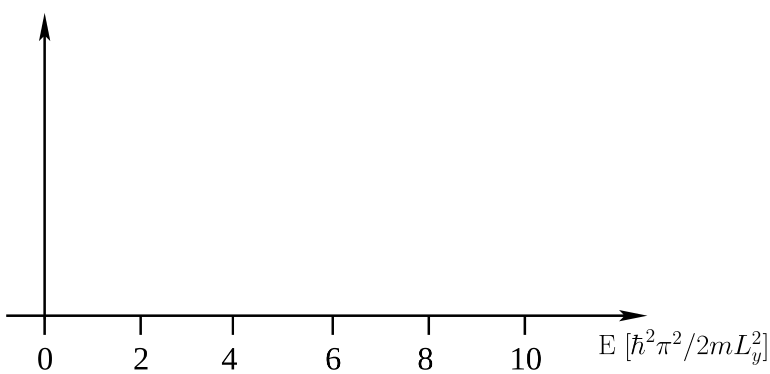
(b) In your homework (Simon #3.1) you derived the static magnetoconductivity tensor

$$\begin{pmatrix} j_x \\ j_y \end{pmatrix} = \frac{\sigma_0}{1+(\omega_c\tau)^2}\begin{pmatrix} 1 & -\omega_c\tau \\ \omega_c\tau & 1 \end{pmatrix}\begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

where the zero field conductivity $\sigma_0 = ne^2\tau/m$ and the cyclotron frequency $\omega_c = eB/m$. Compare the high field limits of σ_{xx} and σ_{eff}. Comment on the result as $B \to \infty$.

2. Quantum channel density of states.

Consider a two-dimensional metal strip of width L_y which is small relative to the electron mean free path. Sketch the electronic density of states on the axes below, and briefly justify your answer.



3. Elastic waves in cubic crystals

(a) Elastic waves propagating with wavevector $\mathbf{K}$ in the cubic $[hkl]$ direction (*i.e.* $\mathbf{K}$ is in the direction $h\hat{x} + k\hat{y} + l\hat{z}$, with $\hat{x}$, $\hat{y}$, and $\hat{z}$ perpendicular to the faces of the cube) exhibit one longitudinally polarized mode and two transversely polarized modes only for special values of $[hkl]$. State these values and briefly justify your assertion without calculation.

(b) For what values of $[hkl]$ do the two transverse sound velocities equal each other? Briefly justify your answer without calculation.

(c) Elastic distortions in cubic crystals create forces with z-component

$$f_z = C_{11}\frac{\partial^2 u_z}{\partial z^2} + C_{12}\left(\frac{\partial^2 u_x}{\partial z \partial x} + \frac{\partial^2 u_y}{\partial z \partial y}\right) + C_{44}\left(\frac{\partial^2 u_x}{\partial z \partial x} + \frac{\partial^2 u_y}{\partial z \partial y} + \frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2}\right)$$

with similar expressions for the x- and y-components. Write down a function $\mathbf{u}(\mathbf{r}, t)$ describing a wave propagating in the [110] direction with displacement in the z direction. Denote the wavevector of this wave by $\mathbf{K}$, the amplitude by A, and the frequency of the wave by ω. Is this wave longitudinal, or is it transverse? What is the speed of propagation?