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## 33-448 Solid State Physics

Midterm \#2 Wednesday, Nov. 152017

1. Consider the linear chain illustrated below, in which tightly bound molecules (spring constant $C_{2}$ ) are weakly bound with spring constant $C_{1}$. All atoms are identical with mass $M$. The lattice constant is $a$. $u_{s}$ and $v_{s}$ are the displacements of atoms in molecule $\# s$. This serves as a model for a crystal of diatomic molecules such as solid $\mathrm{H}_{2}$.

(a) Write down Newton's equations of motion for $u_{s}$ and for $v_{s}$.
(b) At a given wavenumber $K$, how many modes (frequencies) do you expect (and why)?
(c) Determine the frequencies for $K=0$. You may answer by solving the equations of motion if you wish, but full credit will be given only for nonalgebraic solutions based on symmetries.
(d) Determine the frequencies for $K=\pi / a$. You may answer by solving the equations of motion if you wish, but full credit will be given only for nonalgebraic solutions based on symmetries.
(e) Determine the low frequency sound speed (Hint: springs in series obey $1 / C=1 / C_{1}+1 / C_{2}$ ).
2. Barium Oxide and Magnesium Oxide both crystalize into the NaCl structure shown below, consisting of two interpenetrating FCC latices. You will need to know the atomic numbers of Oxygen $(Z=8)$, Magnesium $(Z=12)$ and Barium $(Z=56)$ and the fact that two electrons transfer from barium or magnesium to oxygen, resulting in $\mathrm{Ba}^{++} \mathrm{O}^{--}$or $\mathrm{Mg}^{++} \mathrm{O}^{--}$. Let $\mathbf{k}=\frac{2 \pi}{a}(h \hat{x}+k \hat{y}+l \hat{z})$.

(a) Determine the structure factor $S^{\mathrm{FCC}}(\mathbf{k})$ for an FCC crystal and state any selection rules. Recall that the atoms (with form factor $f$ ) are located at

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(0,0,0), \frac{a}{2}(\hat{y}+\hat{z}), \frac{a}{2}(\hat{z}+\hat{x}), \frac{a}{2}(\hat{x}+\hat{y})
$$

in the conventional cubic unit cell.
(b) Determine the form factors of $\mathrm{Ba}^{++}, \mathrm{O}^{--}$and $\mathrm{Mg}^{++}$.
(c) Recall that the structure factor of a lattice with a basis is the product of the lattice structure factor times the basis structure factor. Calculate the structure factor of Barium Oxide when
(i) $h, k, l$ are all even:
(ii) $h, k, l$ are all odd:
(iii) $h, k, l$ are mixed even and odd:
(iv) The crystal is Magnesium Oxide instead of Barium Oxide and $h, k, l$ are all odd:

