NAME:

33-448 Solid State Physics Midterm #2 Wednesday, Nov. 15 2017

1. Consider the linear chain illustrated below, in which tightly bound molecules (spring constant C_2) are weakly bound with spring constant C_1 . All atoms are identical with mass M. The lattice constant is a. u_s and v_s are the displacements of atoms in molecule #s. This serves as a model for a crystal of diatomic molecules such as solid H₂.



(a) Write down Newton's equations of motion for u_s and for v_s .

(b) At a given wavenumber K, how many modes (frequencies) do you expect (and why)?

(c) Determine the frequencies for K = 0. You may answer by solving the equations of motion if you wish, but full credit will be given only for **nonalgebraic** solutions based on symmetries.

(d) Determine the frequencies for $K = \pi/a$. You may answer by solving the equations of motion if you wish, but full credit will be given only for **nonalgebraic** solutions based on symmetries.

(e) Determine the low frequency sound speed (Hint: springs in series obey $1/C = 1/C_1 + 1/C_2$).

2. Barium Oxide and Magnesium Oxide both crystalize into the NaCl structure shown below, consisting of two interpenetrating FCC latices. You will need to know the atomic numbers of Oxygen (Z = 8), Magnesium (Z = 12) and Barium (Z = 56) and the fact that two electrons transfer from barium or magnesium to oxygen, resulting in Ba⁺⁺O⁻⁻ or Mg⁺⁺O⁻⁻. Let $\mathbf{k} = \frac{2\pi}{a}(h\hat{x} + k\hat{y} + l\hat{z}).$



(a) Determine the structure factor $S^{\text{FCC}}(\mathbf{k})$ for an FCC crystal and state any selection rules. Recall that the atoms (with form factor f) are located at

$$(0,0,0), \ \frac{a}{2}(\hat{y}+\hat{z}), \ \frac{a}{2}(\hat{z}+\hat{x}), \ \frac{a}{2}(\hat{x}+\hat{y})$$

in the conventional cubic unit cell.

(b) Determine the form factors of $\mathrm{Ba^{++},\,O^{--}}$ and $\mathrm{Mg^{++}}.$

(c) Recall that the structure factor of a lattice with a basis is the product of the lattice structure factor times the basis structure factor. Calculate the structure factor of Barium Oxide when

(i) h, k, l are all even:

(*ii*) h, k, l are all odd:

(iii) h, k, l are mixed even and odd:

(iv) The crystal is Magnesium Oxide instead of Barium Oxide and h, k, l are all odd: