NAME: _____

33-783 Solid State Physics   Final Exam   Tuesday, Dec. 12, 2017

Please read each question carefully before answering. Do not do unnecessary work - it will waste time and might cost you points. Feel free to take equations or other facts from the book or your notes, but only if they are directly relevant and useful. Be sure to attempt every part of every question, as many parts can be solved independently of others.

1. Faculty course evaluation Have you completed your FCE, or will you do so? Yes (1 point)/No (0 points)

2. Magnetoconductivity and Hall effect

(a) A p-type semiconductor with effective mass m is placed in a magnetic field $\mathbf{B} = (0, 0, B_z)$ with $B_z > 0$. In which direction do the charge carriers rotate when viewed from above, clockwise or counterclockwise?

(b) The semiconductor is now placed in an electric field $\mathbf{E} = (E_x, E_y, 0)$. Determine the resulting steady-state current density $\mathbf{j}$. Please express your result in terms of the Drude conductivity $\sigma_0 = pe^2\tau/m$,the cyclotron frequency $\omega_c = eB_z/m$ and the relaxation time τ. The matrix relating $\mathbf{j}$ to $\mathbf{E}$ is known as the magnetoconductivity tensor $\boldsymbol{\sigma}$. You only need concern yourself with the x and y components of $\mathbf{j}$ and $\mathbf{E}$.

(c) In the Hall geometry, a current j_x is passed through the sample and an electric field E_y results. What is the physical origin of E_y? Derive the value of E_y as a function of j_x, B, and any other relevant quantities.

3. Brillouin zones and Fermi surfaces
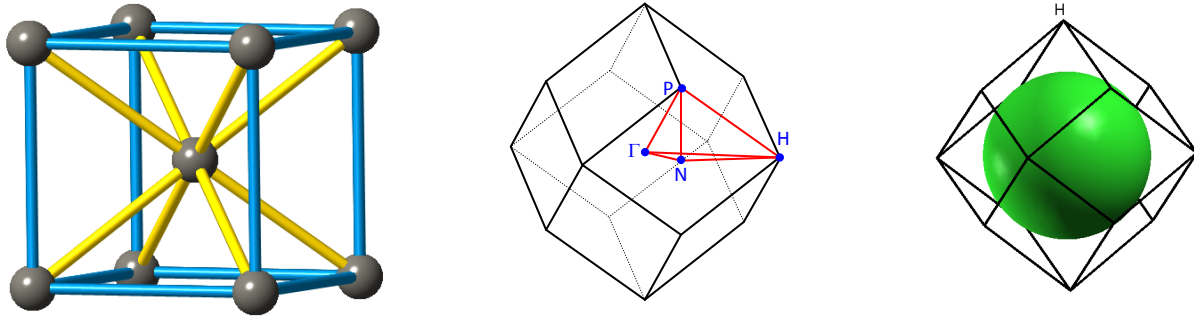
(a) Examine the body centered cubic (BCC) lattice and first Brillouin zone shown above, left. The special symmetry points H and N sit at $\mathbf{k}_H = \mathbf{G}_H/2$ and $\mathbf{k}_N = \mathbf{G}_N/2$, where $\mathbf{G}_H$ and $\mathbf{G}_N$ are reciprocal lattice vectors. State the Miller indices of $\mathbf{G}_H$ and $\mathbf{G}_N$ and briefly justify your assertion. Be sure to use the conventional (simple cubic) indexing, as is usual for BCC structures.

(b) The special point P is located on a $3x$ symmetry axis at the point $\mathbf{k}_P = \frac{\pi}{a}(1, 1, 1)$ (a is the simple cubic lattice constant). Is there diffraction at the reciprocal space position $2\mathbf{k}_P$? What aspect of the BCC structure creates or forbids diffraction at this position?

(c) Now consider the Fermi surface illustrated on the right. Is the material a metal, or an insulator? What is the valence? Briefly justify your conclusions.

(d) The H-point (sharp corner) sits a distance 1.91 Å^{-1} away from the Γ-point. By visual inspection of the figure, estimate the Fermi wavenumber k_F.

(e) In order to calculate the electron dispersion relation $E(\mathbf{k})$ of this material, which would be a more accurate starting point, a tight-binding model or the nearly-free electron approximation? Why?

4. Peierls transition

An infinite one dimensional chain of monovalent atoms has bonds of length a and spring constant C. The electronic band structure can be described by a tight-binding model with onsite energy $\epsilon_0 = 0$ and nearest neighbor electron hopping amplitude (matrix element) $-t$ (with $t > 0$).
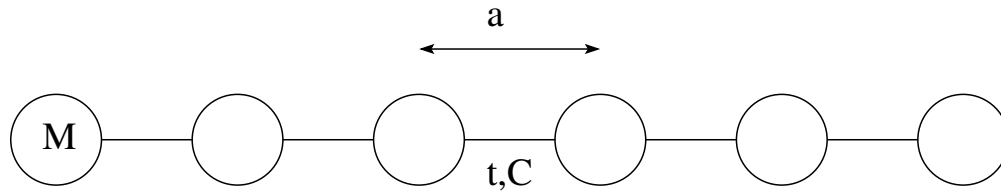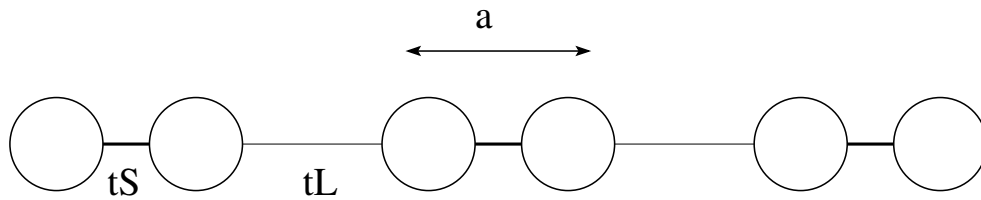

(a) Sketch the electron dispersion relation on the axes provided, and label the Brillouin zone boundaries, the Fermi energy E_F, and the Fermi wavevector k_F.
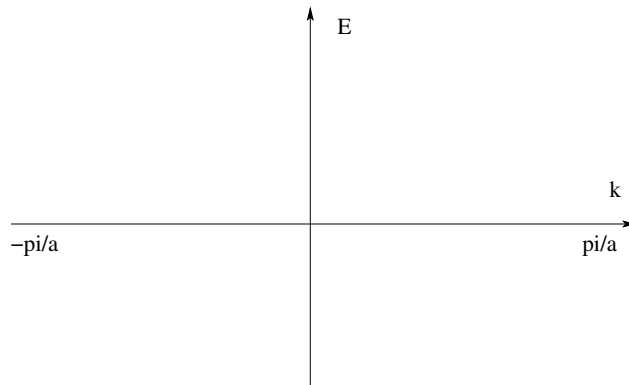

(b) Calculate the band energy E_b, defined as the energy per atom.

(c) The chain distorts causing alternating displacements of $\pm\delta$ as shown. What is the wavenumber K of this distortion? What is the elastic energy per atom E_d given the distortion amplitude δ?

(d) Due to the displacement, the bond lengths alternate short and long, with hopping amplitudes $t_{S,L} = t \pm \delta$. Sketch the electron dispersion relation on the axes provided, and label the Brillouin zone boundaries, the Fermi energy E_F, and the Fermi wavevector k_F. Your sketch should be accurate, but need not be exact.
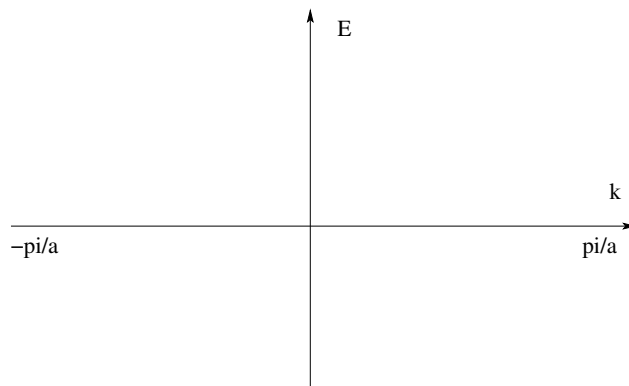
(e) Which energy had the greater change in magnitude ($|E_b|$ or $|E_d|$) for small δ? Will the chain spontaneously distort? Briefly justify your answer. Hint, the band gap is linear in δ.

(f) (Hard) Calculate the band gap E_g exactly. Hint: this easiest if you can *guess* the eigenstates at the band edges.