Kohn anomaly (adapted from Kittel #4.4)

Consider a linear chain with long-range force constants between atoms s and $s + p$ of the form $C_p = A \sin(pk_0 a)/(pa)$, where A and k_0 are constants and p runs over all integers. This form of arises in metals, because interatomic interactions are mediated by the Fermi gas of electrons through ripples in the electron density (Friedel oscillations at frequency $k_0 = 2k_F$) caused by the ion-electron interaction.

(a) Using methods similar to Ashcroft&Mermin #22.1 (also see Kittel chapter 4) derive expressions for $\omega^2(k)$ and $\partial \omega^2/\partial k$, with k the phonon wavenumber.

(b) Show that $\partial \omega^2/\partial k$ diverges at $k = k_0$. Sketch a graph of $\omega(k)$.