Quantum of thermal conductance (adapted from Kittel #18.6)

Recall the Landauer formula for electrical conductance, $G = (2e^2/h)\mathcal{T}$, with $\mathcal{T}$ the transmission coefficient for electrons through the channel (*i.e.* the probability that an electron that enters the channel will pass through). You will derive a similar result for the thermal conductance of phonons.

(a) The dispersion relation for longitudinal phonons is $\omega = vK$, with v the speed of sound. Derive the low temperature heat capacity, $C = 2\pi^2 L {k_\mathrm{B}}^2 T/3hv$, of a channel of length L.

(b) Apply a temperature drop $\Delta T = T_H - T_L$ between the ends of the channel. Considering the heat carried by phonons, and their transmission coefficients $\mathcal{T}$ (assumed independent of ω), derive the thermal conductance (defined as rate of energy transport divided by ΔT). It will help to consider the fluxes of heat separately from reservoirs at each end of the channel. Note the result $G = \pi^2 {k_\mathrm{B}}^2 T\mathcal{T}/3h$ is universal in that it does not depend on the material property v (speed of sound), nor does it depend on the channel length L.

(c) Consider a transverse flexural mode with dispersion relation due to bending of $\omega = cK^2$, where c is a constant related to the bending stiffness, and show that again the thermal conductance is independent of the material property c and channel length L. You may carry this out by repeating parts (a) and (b), or instead find a simple mathematical argument. Note: energy is transported at the *group* velocity $v = \mathrm{d}\omega/\mathrm{d}K$.