NAME: _____

33-342 Thermal Physics II Midterm #1 Wednesday, February 14, 2018

This problem concerns a spin-1 ($\sigma_z = -1, 0, +1$) particle subject to a crystal field D and a magnetic field B in the z direction. The system is defined by its Hamiltonian

$$H = -B\sigma_z + D{\sigma_z}^2.$$

Notice that the crystal field favors the state $\sigma_z = 0$, *i.e.* the spin in the xy-plane. Please take careful note of the $\pm$ signs in the Hamiltonian.

1. Write down the partition function Z and the free energy F.

2. Work out the statistical average of the magnetic moment, $M \equiv \langle \sigma_z \rangle$.

3. Compare your average magnetic moment with the derivative $\partial F/\partial B$.

4. A formal identity exists relating $Q \equiv \langle {\sigma_z}^2 \rangle$ to a derivative of free energy. What is this derivative? Justify your conclusion.

5. The derivative $R \equiv \partial M/\partial D$ can be expressed in terms of statistical averages. What is this expression? Give a formal expression involving thermal averages of the type $\langle A \rangle$ for suitable observables A. Note that **you are not asked to evaluate the averages**, only to give the formal expression in terms of $\langle A \rangle$.

6. Using your physical intuition, and assuming $B > 0$, do you expect R to be positive, or negative? Briefly explain, in words.