NAME: ____________________________________

33-342 Thermal Physics II   Midterm #2   Friday, April 6, 2018

Fluctuations of particles and holes

1. The occupancy fluctuation for a quantum state is defined as

$$\sigma_n^2 = \langle n^2 \rangle - \langle n \rangle^2.$$

For fermions this can be simply expressed in terms of the average occupation probability $\langle n \rangle$ and the average vacancy probability $\langle h \rangle = 1 - \langle n \rangle$ (here h stands for occupation by a "hole"). What is the expression? *Hint:* What can you say about the value of n^2 for a fermion?

2. Sketch σ_n^2 as a function of $\langle n \rangle$ over the full range of $\langle n \rangle$. Justify the shape and special values of this function on physical grounds.

3. In a gas of fermions with chemical potential μ, consider a state of energy E. The probability to occupy this state is

$$\langle n \rangle = \frac{1}{e^{\beta(E-\mu)}+1}.$$

Rewrite the vacancy probability $1 - \langle n \rangle$ as the probability $\langle h \rangle$ to occupy a hole state, and relate the energy E_h and chemical potential μ_h of the hole to the corresponding properties of the particle. Is the hole a fermion, or a boson?

4. Compute Deserno's "fillability" $\chi \equiv \partial\langle n \rangle/\partial\mu$. Do you see a connection between this result and your expression for σ_n^2 found in part 1? The relationship is a simple example of a general connection between fluctuations and susceptibilities.

5. Does a similar relation hold for bosons? *Hint:* Recall that $\sigma_n^2 = \langle n \rangle (1 + \langle n \rangle)$ for bosons.