NAME: ______________________________

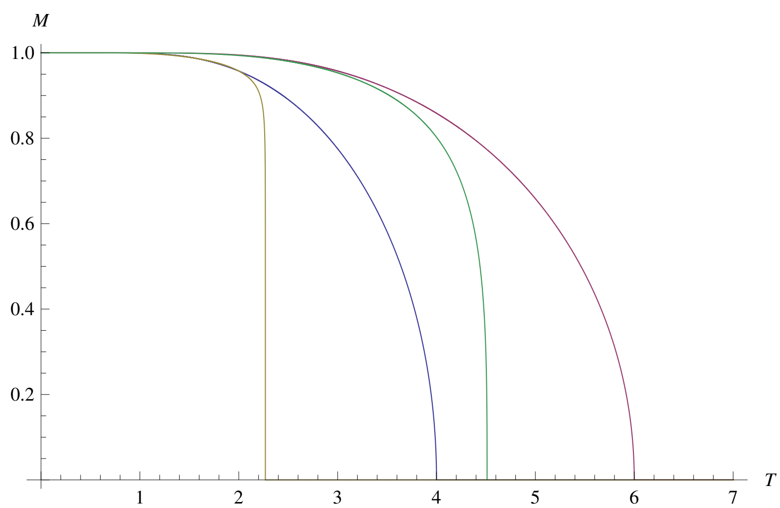
33-342 Thermal Physics II Final Exam Friday, May 4, 2018

1. Have you completed your FCE, or will you do so? Yes (1 point)/No (0 points)

The following questions address the Ising model with Hamiltonian

$$H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j - h \sum_i \sigma_i .$$

2. The following figure illustrates the exact zero-field magnetization $M(T, h = 0)$ for the Ising model on a square lattice in 2D and a simple cubic lattice in 3D, and also the mean field theory solution for each lattice. Units are chosen so that $J = k_{\mathrm{B}} = 1$. Label each of the curves and briefly justify your claims, addressing both the critical temperatures T_c and the critical exponents β.



3. Consider the Ising model on an equilateral triangle with $N = 3$ spins at the vertices. Take the antiferromagnetic case with $J < 0$. Describe the ground state and state the entropy at $T = 0$.

4. Recall that the low temperature series for the square lattice Ising model with N sites and magnetic field $h = 0$ is

$$Z_0 = 2e^{2N\beta J}\{1 + Ne^{-8\beta J} + \cdots\}.$$

In the following you will keep only terms of similar order to the above. Each part that follows depends on your previous result, so **be very careful!** If you cannot complete an early step, you can introduce a symbol to represent your answer then proceed with the later steps.

(a) Explain briefly the meaning or origin of the leading factor of 2, the exponential prefactor $\exp(2N\beta J)$, and the term $N\exp(-8\beta J)$ in the series expansion.

(b) Recalculate the expansion of Z for the case of positive magnetic field, $h > 0$.

(c) Obtain a series expansion for the free energy. *Please work in the limit of large size N, such that $N\beta h \gg 1$.*

(d) Calculate the magnetization M.

(e) Determine the zero-field susceptibility defined as

$$\chi_0 \equiv \lim_{h \to 0^+} \frac{\partial M}{\partial h}.$$

Sketch your result for $\chi_0(T)$ and discuss its approach to its low temperature limit.