NAME: ________________________________

33-342 Thermal Physics II Midterm #1 Wednesday, March 6, 2019

The following problem is adapted from Swendsen #24.1. Note that part 6 is independent of parts 2-5. Throughout this problem dimensionality and power laws are more important than constant prefactors.

The universe has collapsed and you are stuck in a d-dimensional world of volume $V = L^d$, kept warm by a d-dimensional black-body at temperature T. To make matters worse the photon dispersion relation became nonlinear, so that $\omega = ck^s$, with c and s positive constants. Luckily, no other physical laws have changed. For example, the occupation of a state of energy $E = \hbar\omega$ is still given by

$$\langle n_\omega \rangle = \frac{1}{e^{\beta\hbar\omega} - 1}.$$

To estimate your chances of survival you wish to calculate the available energy.

1. Calculate the photon density of states $D(\omega)$. *Hint:* You have learned a few ways to approach this problem. If you take the route of counting modes below a given frequency, it may help with some later parts.

2. Write down a formal expression for the total energy U available in this world, neglecting the zero point energy. Your expression may be written in the form of an integral. Make sure that every quantity in your expression is defined but **do not** bother to evaluate the integral.

3. U is proportional to a power of the temperature T, *i.e.* $U \sim T^p$. Express p in terms of d and s. *Hint:* a suitable change of variables can take the temperature dependence outside the integral.

4. Give a brief intuitive explanation for the value of p in terms of the states that are excited and their energy content.

5. Check that your answer is correct for $d = 3$ and $s = 1$ (*i.e.* ordinary photons in ordinary space).

6. Imagine that you have a d-dimensional solid body of N atoms in volume $V = L^d$. The atoms interact with a strange potential leading to the dispersion relation $\omega = ck^s$. (a) Write down a formal expression that determines the Debye frequency ω_D. (b) The Debye frequency varies with density as $(N/V)^q$ for some power q. Determine q and check that it has the expected value for $d = 3$ and $s = 1$ (*i.e.* ordinary phonons).