NAME: ________________________________

33-342 Thermal Physics II Midterm #2 Tuesday, April 2, 2019

1. Recall that heat capacity is the rate at which a material gains energy as the temperature rises. Answer the following questions in a few brief sentences using only physical reasoning without any equations.

(a) Which has the greater heat capacity at low temperature, a Fermi gas or a Bose gas? Both have the same number of particles and the same masses in the same volumes.

(b) The heat capacity of an ideal gas at high temperature is $3k_{\mathrm{B}}/2$, while the heat capacity of a harmonic solid is $3k_{\mathrm{B}}$ at high temperature. Why is the heat capacity greater for a solid than for an ideal gas?

2. A donor state is the state of an electron bound to a donor ion in a semiconductor (*e.g.* an electron bound to a phosphorous ion in a doped semiconductor). The energy of this state, E_d, lies in the gap, slightly below the conduction band minimum, $E_d \lesssim E_c$. A donor state can be empty, or it can be occupied by a spin up electron or by a spin down electron. Because of Coulomb repulsion it cannot be occupied by a spin up *and* and spin down electron at the same time.

(a) Write down the partition function $\mathcal{Z}_d$ for a single donor state. Don't forget to include the chemical potential μ of the electron.

(b) Determine the mean occupation $\langle n_d \rangle$ of the donor state.

(c) As a simplified model for a semiconductor, assume there are N_d donor states all at energy E_d, and N_c conduction states, all at a common energy E_c, with $E_d < E_c$. Express the total number of electrons N_e in terms of the quantities N_d, N_c, E_d, E_c, and μ.

(d) (hard) The total number of electrons, $N_e = N_d$, is shared among the donor and conduction states. Determine the chemical potential $\mu(T)$ at low temperatures. *Hint:* this problem is closely related to Swendsen #28.1. Your solution will be aided if you use the identity

$$\frac{1}{x/a+1} + \frac{1}{a/x+1} = 1.$$