NAME: ____

33-342 Thermal Physics II   Final Exam   Friday, May 10, 2019

1. Have you completed your FCE, or will you do so? Yes (1 point)/No (0 points)

2. Consider the relativistic energy-momentum relation, $E = \sqrt{p^2c^2 + m^2c^4}$, for a particle of rest mass m.

(a) In an $L \times L \times L$ box, calculate the number of states, $\mathcal{N}(E)$, of energy less than E. You may simplify your algebra by taking units in which $c = \hbar = 1$.

(b) Determine the density of states $D(E)$.

(c) Sketch $D(E)$ on the interval $E \in (10m, 1000m)$. Your sketch should approximate a familiar density of states function. What is that function, and why (intuitively) do you obtain it? *Hint:* you need to find a power law in the high energy limit $E \gg m$.

(d) No states have energy less than m. Sketch $D(E)$ on the interval $E \in (m, 1.01m)$. Your sketch should approximate a familiar density of states function. What is that function, and why (intuitively) do you obtain it? *Hint:* you need to find a power law in the low energy limit of $E \gtrsim m$.

3. A one-dimensional spin-1 chain with periodic boundary conditions has the Hamiltonian

$$H = \sum_{j=1}^{N} \left\{ -J\sigma_j\sigma_{j+1} - \frac{h}{2}(\sigma_j + \sigma_{j+1}) - \frac{D}{2}(\sigma_j^2 + \sigma_{j+1}^2) \right\}$$

(a) Write down the transfer matrix $T(\sigma_j, \sigma_{j+1})$. Be sure to specify the basis that you write your matrix in.

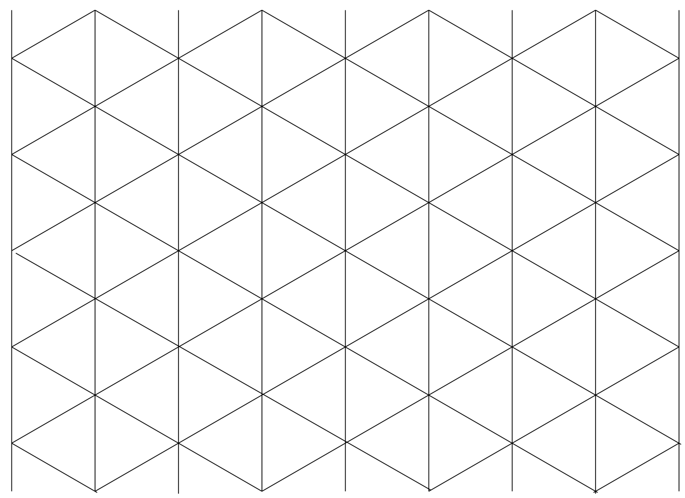
(b) For $J > 0$ the transfer matrix has three eigenvalues, $0 < \lambda_1 < \lambda_2 < \lambda_3$. In terms of these eigenvalues express the free energy per spin, $f = F/N$, in the limit of large N.

(c) The average

$$Q \equiv \langle \sum_j \sigma_j^2 \rangle$$

can be formally expressed as a derivative of the free energy. Write down this formal expression and briefly justify it.

4. Consider a nearest-neighbor Ising model with N Ising spins ($\sigma_i = \pm 1$) at the vertices of a triangular lattice with periodic boundary conditions (see below). The Hamiltonian $H = -J\sum_{\langle ij\rangle} \sigma_i \sigma_j$ has ferromagnetic coupling ($J > 0$).



(a) Determine the first two terms in a low temperature series for the partition function.

(b) Determine the first two terms in a high temperature series for the partition function. You may wish to recall the identity $\exp(\beta J \sigma\sigma') = \cosh(\beta J)(1 + v\sigma\sigma')$, with the reduced coupling strength $v = \tanh(\beta J)$.

(c) Derive the self-consistent equation for magnetization according to mean-field theory.

(d) Think about the ground state in the antiferromagnetic case ($J < 0$). How would you describe the ground state configuration(s)? Feel free to illustrate using the lattice given.