Sommerfeld Examples (Fermi gas at low T)

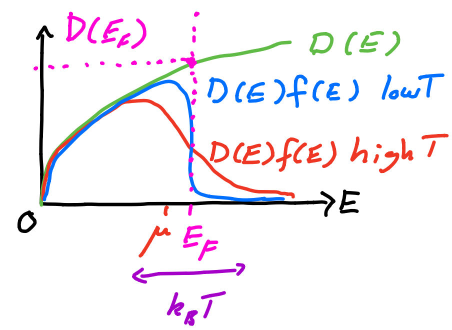
Chemical potential $N = \int_0^\infty dE\, D(E) f_{T\mu}(E) = \int_0^\mu dE\, D(E) + \frac{\pi^2}{6} D'(\mu)(k_B T)^2 + \cdots$

by definition of $E_F \equiv \lim_{T\to 0} \mu(T,N) \Rightarrow N = \int_0^{E_F} dE\, D(E) \approx \int_0^\mu dE\, D(E) + (E_F - \mu) D(E_F)$

$$\therefore \mu \approx E_F - \frac{\pi^2}{6}(k_B T)^2 D'(E_F)/D(E_F)$$

As $T\uparrow$, μ moves towards lower $D(E)$ to keep N constant

$\mu - E_F = O(T^2)$ slowly varying provided $D(E_F) \neq 0 \Rightarrow \mu \approx E_F$ for metals



Energy $U = \int_0^\infty dE\, E\, D(E) f_{T\mu}(E)$

$$\approx U_0 + (\mu - E_F) E_F D(E_F) + \frac{\pi^2}{6} \frac{d}{dE}(E D(E))\Big|_{E_F} (k_B T)^2$$

$U_0 = \int_0^{E_F} dE\, E\, D(E)$; $(\mu - E_F) \sim -E_F D'(E_F)$; $\frac{d}{dE}(E D(E)) = D(E_F) + E_F D'(E_F)$; $E_F D'(E_F)$ cancels

$$\therefore U \approx U_0 + \frac{\pi^2}{6} D(E_F)(k_B T)^2$$

Heat Capacity

$$C = \frac{\partial U}{\partial T}\Big|_{NV} = \frac{\pi^2}{3} D(E_F) k_B^2 T$$

$C \sim T^1$ compared to T^3 for phonons

Note: only electrons near E_F contribute -

$E < E_F$ frozen out by Pauli

$E > E_F$ are empty states