General Case $J \neq 0$ $h \neq 0$ best solved with P.B.C.

$$H = -J\sum_{j=1}^{N} \sigma_j \sigma_{j+1} - h\sum_{j=1}^{N}\sigma_j = -\sum_{j=1}^{N}\left\{J\sigma_j\sigma_{j+1} + \frac{h}{2}(\sigma_j + \sigma_{j+1})\right\}$$

↑ more symmetric!

$$Z = \sum_{\{\sigma_j\}} e^{-\beta H} = \sum_{\{\sigma_j\}} \prod_j \underbrace{e^{\beta\{J\sigma_j\sigma_{j+1} + h(\sigma_j + \sigma_{j+1})\}}}$$

$$T(\sigma_j, \sigma_{j+1}) \longrightarrow \overleftrightarrow{T} = \begin{pmatrix} e^{\beta(J+h)} & e^{-\beta J} \\ e^{-\beta J} & e^{\beta(J-h)} \end{pmatrix}$$ "Transfer matrix"

$$Z = \sum_{\sigma_1}\sum_{\sigma_2}\cdots\sum_{\sigma_N} \underbrace{T(\sigma_1\sigma_2)\,T(\sigma_2\sigma_3)}\cdots T(\sigma_N\sigma_{N+1})$$

$\sigma_{N+1} \equiv \sigma_1$

$$\sum_{\sigma_2} T(\sigma_1\sigma_2)T(\sigma_2\sigma_3) = T^2(\sigma_1\sigma_3)$$

$$\therefore Z = \sum_{\sigma_1} T^N(\sigma_1, \sigma_1) = Tr\,\overleftrightarrow{T}^N$$

Facts about traces:

$Tr\,M = \lambda_1 + \lambda_2 + \cdots$

because $M = UDU^{-1}$

$$D = \begin{pmatrix} \lambda_1 & 0 & \\ 0 & \lambda_2 & \\ & & \ddots \end{pmatrix} \quad D^N = \begin{pmatrix} \lambda_1^N & 0 & \\ 0 & \lambda_2^N & \\ & & \ddots \end{pmatrix}$$

$\therefore Tr\,M^N = \lambda_1^N + \lambda_2^N + \cdots$

Eigenvalues of $\overleftrightarrow{T}$? symmetric $\Rightarrow$ real

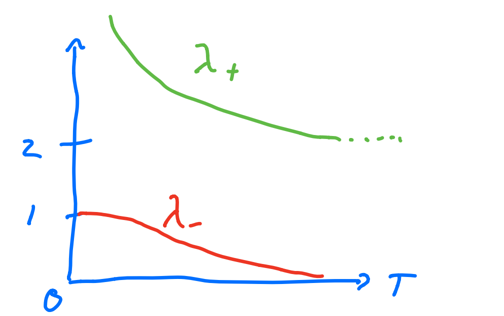
$\det(\overleftrightarrow{T} - \lambda\overleftrightarrow{1}) = 0 \Rightarrow$ quadratic eq'n

$$\lambda = e^{\beta J}\cosh(\beta h) \pm \sqrt{\underbrace{e^{2\beta J}\cosh^2(\beta h) - 2\sinh(2\beta J)}_{\equiv D \text{ "discriminant"}}}$$

$\equiv \lambda_\pm$

$$\lambda_+ \longrightarrow \begin{cases} 2 \text{ as } T \to \infty \\ 2e^{\beta J}\cosh(\beta h) \to \infty \text{ as } T \to 0 \end{cases}$$

$$\lambda_- \longrightarrow \begin{cases} 2J/kT \to 0 \text{ as } T \to \infty \\ 1 \text{ as } T \to 0 \end{cases}$$



$$Z = \lambda_+^N + \lambda_-^N = \lambda_+^N(1 + (\lambda_-/\lambda_+)^N) \to \lambda_+^N \text{ for large } N$$

$$F = -kT\ln Z \to -NkT\ln\lambda_+$$

$$M = -\frac{\partial F}{\partial h} = Ne^{J/T}\sinh(h/T)/\sqrt{D}$$

$$\chi = \frac{\partial M}{\partial h} = \frac{1}{T}e^{-J/T}/\sqrt{D}$$