Transfer matrix solution of the 1D Ising model.

Recall the transfer matrix solution of the 1D Ising model presented in

$$Z = \lambda_+^N + \lambda_-^N$$

This version best for small h

with

2sinh(2βJ)

$$\begin{aligned}\lambda_\pm &= e^{\beta J}\cosh(\beta h) \pm \sqrt{e^{2\beta J}\cosh^2(\beta h) - 2\sinh\beta J} \\ &= e^{\beta J}\cosh(\beta h) \pm \sqrt{e^{2\beta J}\sinh^2(\beta h) + e^{-2\beta J}}\end{aligned}$$

being the eigenvalues of the transfer matrix

$h = 0$, $\sinh\beta h = 0$, $\cosh\beta h = 1$

$$T = \begin{pmatrix} e^{\beta(J+h)} & e^{-\beta J} \\ e^{-\beta J} & e^{\beta(J-h)} \end{pmatrix}$$

$$T = \begin{pmatrix} e^{\beta h} & e^{-\beta h} \\ e^{-\beta h} & e^{\beta h} \end{pmatrix}$$

$$\lambda_\pm = e^{\beta J} \pm e^{-\beta J}$$

1. Find simple expressions for λ_+ and λ_- for the case $h = 0$. $= \underset{\lambda_+}{2\cosh\beta J},\ \underset{\lambda_-}{2\sinh\beta J}$
2. Take the limit of large N and expand the free energy F up to second order in the magnetic field h. $\lambda_+ > \lambda_- \Rightarrow F = -NkT\ln\lambda_+ = -NkT\ln(2\cosh\beta J) - (N/2kT)e^{2J/kT}h^2$
3. Determine the magnetization per spin

$$M = \frac{-1}{N}\frac{\partial F}{\partial h} = \frac{1}{kT}e^{2\beta J}\cdot h$$

and the susceptibility per spin

$$\chi = \frac{\partial M}{\partial h} = \frac{1}{kT}e^{2\beta J}$$

Interpret your result for χ.

$e^{2\beta J}$ is the enhancement factor ($J>0$) or diminution ($J<0$) due to coupling with 2 neighbors.