Phase Transitions (non-analytic/singular variation of free energy)

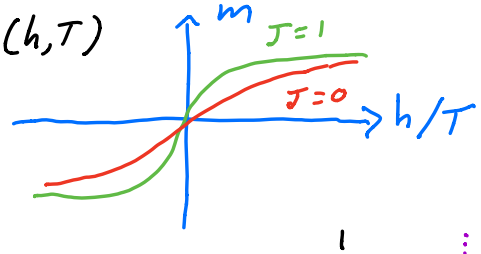
Examples: Bose-Einstein, liquid-gas, liquid-solid, normal-superconductor, para-ferromagnet

They:

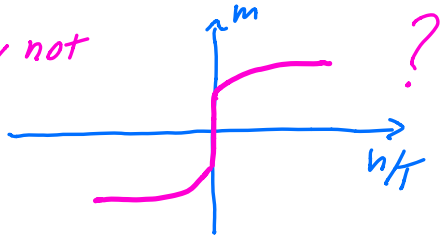
- Usually have "control parameter" $(T, h, p, \ldots)$ to drive transition
- Usually have "order parameter" $(M, \rho, \ldots)$ to distinguish phases
- Never occur in finite size systems ($Z = \sum_{\text{finite \# terms}} e^{-\beta E}$ is an analytic function)
- Sometimes can occur in infinite systems $\left(\text{e.g. } \delta(x) = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty} \cos(nx)\right)$

Example: 1-D Ising chain has no transition

Consider $m(h,T)$



Why not



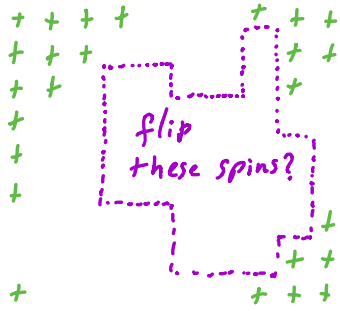
Reason: some configuration:

ΔE finite $(\Delta E = 4J)$

$\Delta S = k_B \ln\left((N-1)(N-2)/2\right) \Rightarrow \Delta E - T\Delta S < 0 \quad \forall T > 0$ as $N \to \infty$

(\# domains to flip)

Example: 2D square lattice of + spins, flip region of perimeter L



$\Delta E = 2JL$, $\Delta S = k_B \ln C^L$ ← C^L = # closed loops length L

$\Delta E/\Delta S = 2J/\ln C \equiv T^*$

$T < T^*$ energy wins ⇒ alignment

$T > T^*$ entropy wins ⇒ disorder

⇑

2 distinct phases

Phase Transitions