Spin-1 Mean Field Theory

$$H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j \qquad \sigma = \pm 1, 0$$

I'll let $h_{eff} \to h$ for simplicity

$$h_{eff} = J z \langle \sigma \rangle \Rightarrow m = \langle \sigma \rangle = \frac{1 \cdot e^{\beta h} + 0 \cdot e^{0} - 1 \cdot e^{-\beta h}}{e^{\beta h} + e^{0} + e^{-\beta h}}$$

$$= \frac{2 \sinh \beta h}{1 + 2 \cosh \beta h}$$

Implicit Equation: $m = \frac{2 \sinh(\beta J z m)}{1 + 2 \cosh(\beta J z m)} = \frac{2}{3}(\beta J z m) - \frac{1}{9}(\beta J z m)^3 + \cdots$

Truncate at $\mathcal{O}(m^3)$ and solve for $m = 0, \pm \sqrt{\frac{6 \beta J z - 9}{(\beta J z)^3}}$

$\therefore \quad k_B T_c = \frac{2}{3} J z \qquad m \sim \sqrt{T_c - T} \Rightarrow \beta = \frac{1}{2}$