Series Analysis

Enting (1994) derived series for c, χ, M up to 76^{th} order

See Ising2DLowTseries.nb mathematica notebook

Ratio Test:

Specific heat $c = 0x^0 + 0x^2 + 16x^4 + 72x^6 + \cdots = \sum_n c_n x^n$

Does series converge? Compare with geometric series.

$$\sum_n (r/\rho)^n = \frac{1}{1 - r/\rho} \quad \text{converges for } |r| < \rho$$

$\therefore$ If $|c_n| < 1/\rho^n$ as $n \to \infty$ then $c(x)$ converges for $|x| < \rho$

If $\lim_{n\to\infty} c_n/c_{n+1} \longrightarrow R$ is a finite value then radius of convergence $\rho = R$

Series for c_n has only even $n \Rightarrow \rho = \lim_{n\to\infty} \sqrt{c_n/c_{n+2}} \stackrel{?}{=} x_c = 0.414$

Critical Exponents:

	Exact
$c = \underbrace{A(T)}_{\text{smooth, positive}}(T_c - T)^{-\alpha}$ as $T \to T_c^-$	$\alpha = 0$
$M = B(T)\,(T_c - T)^{\beta}$	$\beta = 1/8$
$\chi = G(T)\,(T_c - T)^{-\gamma}$	$\gamma = 7/4$

Exponent from Series

$$\ln \chi = \ln G(T) - \gamma \ln(T_c - T) \qquad \frac{\partial \ln \chi}{\partial T} = \underbrace{\frac{G'}{G}}_{\text{smooth, finite}} + \frac{\gamma}{T_c - T}$$

$$(T_c - T)\frac{\partial \ln \chi}{\partial T} = \underbrace{\cancel{(T_c - T)\frac{G'}{G}}}_{0 \text{ as } T \to T_c^-} + \gamma \Rightarrow \gamma = \lim_{T\to T_c^-} (T_c - T)\frac{\partial \ln \chi}{\partial T} \stackrel{?}{=} 7/4$$