Histogram Monte Carlo (Ferrenberg & Swendsen)

$$Z = \int dE\, \Omega(E)\, e^{-\beta E}$$

$$U = \langle E \rangle = \int dE\, \Omega(E) \cdot E \cdot e^{-\beta E} / Z$$

$\Omega(E)$ = density of states

$P(E) = \Omega(E) e^{-\beta E}/Z$

↑ sums if E discrete

Monte Carlo histogram $H(E) \sim P(E)$ unknown constant of proportionality

$$H(E) = G \cdot P(E) \Rightarrow G = \int dE\, H(E)$$

$$U = \int dE \cdot E \cdot H(E) \Big/ \int dE\, H(E)$$

↑ Standard M.C. G cancels out

Invert $P(E) = \frac{1}{Z} \Omega(E) e^{-\beta E}$

$$\Rightarrow \Omega(E) = Z\, P(E) e^{+\beta E} = G' H(E) e^{\beta E} \qquad G' = G Z$$

Given simulation at $\beta_0 = \frac{1}{kT_0} \Rightarrow \frac{1}{G'} \Omega(E)$ over sampled E range

Then $U(T_1) = \int dE \left(\frac{1}{G'} \Omega(E)\right) \cdot E \cdot e^{-\beta_1 E} \Big/ \int dE \left(\frac{1}{G'} \Omega(E)\right) e^{-\beta_1 E}$

$$= \int dE\, H(E) \cdot E \cdot e^{-(\beta_1 - \beta_0)E} \Big/ \int dE\, H(E)\, e^{-(\beta_1 - \beta_0)E}$$

Can also get $C(T_1)$ from $\langle E^2 \rangle - \langle E \rangle^2$

2D Histograms $H(E,M)$ ← MC at T_0 and h_0

$$M(T_1, h_1) = \int dE\, dM \cdot M \cdot H(E,M)\, e^{-[\Delta\beta E - \Delta(\beta h) M]} \Big/ \int dE\, dM\, H(E,M)\, e^{-[\ \ ]}$$

↑ Can get $M(h_1)$ from MC simulation at $h_0 = 0$!