Boltzmann distribution in the canonical ensemble

1. Consider N identical, but distinguishable, particles with eigenstates $\{\alpha\}$. The total energy $E = \sum_\alpha n_\alpha E_\alpha$. Demonstrate that the mean occupation of a specific state α is

$$\langle n_\alpha \rangle = -\frac{1}{\beta}\frac{\partial \ln Z}{\partial E_\alpha}.$$

2. Assuming $Z = z_1^N/N!$, with $z_1 = \sum_\alpha \exp(-\beta E_\alpha)$, find an expression for $\langle n_\alpha \rangle$.

3. In how many ways, $\Omega(n_1, n_2, \dots)$, can the N particles be distributed into so that n_1 are in the first state, n_2 are in the second state, etc.? Notice that $\sum_\alpha n_\alpha = N$.

4. Define

$$\tilde{Z} = \sum_{n_1, n_2, \cdots} \Omega(n_1, n_2, \dots)\, e^{-\beta(n_1 E_1 + n_2 E_2 + \cdots)}$$

and compare with the multinomial expansion of $\left(e^{-\beta E_1} + e^{-\beta E_2} + \cdots\right)^N$ to find a simple expression for $\tilde{Z}$. Compare $\tilde{Z}$ with $Z = z_1^N/N!$ and discuss.