Fermi-Dirac Quantum Gas Problem 2: Specific heat

Using the programs that you've written for quantum gases, carry out the following calculations for Fermi-Dirac statistics.

The Fermi temperature is given by the equation

$$k_B T_F = A \left(\frac{3N}{\pi} \right)^{2/3}$$

It is discussed in the book and will be discussed in class. For this assignment, all you need to know is that it is a characteristic temperature for an ideal gas of fermions.

For $N = 1000$ particles, in a box of size $L = \pi$ (*i.e.* $A = 1$), make three plots of the specific heat as a function of temperature. Let the lowest temperature be $T = 0.01$ for all three plots. Let the highest temperatures be $T/T_F = .001$, $T/T_F = .01$, $T/T_F = 0.1$, and $T/T_F = 2.0$.

1. For $T/T_F < 0.001$, the specific heat is seen to be very small. Why is this true?
2. In a temperature range of about $T = .02T_F$ up to $T = 0.1T_F$, how does the specific heat behave. Explain why this is to be expected.
3. For T much larger than T_F, the specific heat seems to be going to a constant value. Derive the value of this constant. Is the value you derived consistent with your plots?