Correlation functions of the Ising chain

This problem considers the Ising chain with Hamiltonian

$$H = -J\sum_{j=1}^{N}\sigma_j\sigma_{j+1} - \sum_{j=1}^{N} h_j\sigma_j$$

Note that the magnetic field may differ among sites, although we will mainly be interested in the case where all fields vanish, $\vec{h} = \vec{0}$. Assume periodic boundary conditions.

(a) Show that $m_i \equiv \langle\sigma_i\rangle = -\partial F/\partial h_i$

(b) Show that $\chi_{ik} \equiv \partial m_i/\partial h_k = -\partial^2 F/\partial h_i\partial h_k = \beta(\langle\sigma_i\sigma_k\rangle - \langle\sigma_i\rangle\langle\sigma_k\rangle)$.

(c) Show that χ_{ik} is translation invariant when $\vec{h} = \vec{0}$. Hence $\chi_{ik} = g(x)$, where $x = k - i$. Interpret the meaning of $g(x)$.

(d) Show that

$$\sum_{x=-N/2}^{N/2} g(x) = \chi_0,$$

with χ_0 the zero field magnetic susceptibility.