High temperature series for Ising chain.

Recall our high temperature series expansion for the Ising chain,

$$Z = 2^N \cosh^N(\beta J) \cosh^N(\beta h) \left(1 + N u^2 v + \ldots\right)$$

with $u = \tanh(\beta h)$ and $v = \tanh(\beta J)$.

1. Notice the free energy can be written as $F = F_0 + \Delta F$, where $F_0 = -kT \log[2^N \cosh^N(\beta J) \cosh^N(\beta h)]$ and $\Delta F = -kT \log[1 + N u^2 v]$. Expand ΔF up to first order in v. Is the free energy extensive (i.e. proportional to N)?

2. Use this result to calculate the magnetization per spin, $M(J, h)$ accurately up to first order in h.

3. Now calculate the magnetic susceptibility χ. Compare with the exact susceptibility obtained from the transfer matrix.