Transfer matrix solution of the 1D Ising model.

Recall the transfer matrix solution of the 1D Ising model presented in

$$Z = \lambda_+^N + \lambda_-^N$$

with

$$\begin{aligned}\lambda_\pm &= e^{\beta J}\cosh(\beta h) \pm \sqrt{e^{2\beta J}\cosh^2(\beta h) - 2\sinh 2\beta J} \\ &= e^{\beta J}\cosh(\beta h) \pm \sqrt{e^{2\beta J}\sinh^2(\beta h) + e^{-2\beta J}}\end{aligned}$$

being the eigenvalues of the transfer matrix

$$T = \begin{pmatrix} e^{\beta(J+h)} & e^{-\beta J} \\ e^{-\beta J} & e^{\beta(J-h)} \end{pmatrix}$$

1. Find simple expressions for λ_+ and λ_- for the case $h = 0$.
2. Take the limit of large N and expand the free energy F up to second order in the magnetic field h.
3. Determine the magnetization per spin

   $$M = \frac{-1}{N}\frac{\partial F}{\partial h}$$

   and the susceptibility per spin

   $$\chi = \frac{\partial M}{\partial h}.$$

   Interpret your result for χ.