Quantum Mechanics Review: homework #1

1. If \( \{|m\rangle, m \in \mathbb{N} \} \) is a complete set of eigenstates of a Hamiltonian \( H \) with nondegenerate energies \( E_m \), and \( |\psi\rangle = \sum_m c_m |m\rangle \), what is the probability that the system has energy \( E_m \) for some particular \( m = n \)? What is the average energy of the system?

2. Give the eigenstates and their energies for a spin 1/2 particle in an applied magnetic field, \( B = B\hat{z} \).

3. For a spin 1 particle the eigenstates of \( S_z \) are \( \{|+\hat{z}\rangle, |0\hat{z}\rangle, |-\hat{z}\rangle \} \) with eigenvalues, respectively, \( +\hbar \), 0, and \( -\hbar \). However, the action of \( S_x \) is:

\[
S_x|+\hat{z}\rangle = (\hbar/\sqrt{2})|0\hat{z}\rangle, \quad S_x|0\hat{z}\rangle = (\hbar/\sqrt{2})(|+\hat{z}\rangle + |-\hat{z}\rangle), \quad S_x|-\hat{z}\rangle = (\hbar/\sqrt{2})|0\hat{z}\rangle.
\]

(i) Represent \( S_z \) and \( S_x \) as \( 3 \times 3 \) matrices in the \( \hat{z} \) basis.

(ii) Express the ket-vector \( |+\hat{x}\rangle \) as a linear combination in the \( \hat{z} \) basis.

(iii) What is the inner product \( \langle 0\hat{z}|+\hat{x}\rangle \)?