Quantum Mechanics Review: homework #1

1. If $\{|m\rangle, m \in \mathbb{N}\}$ is a complete set of eigenstates of a Hamiltonian H with nondegenerate energies E_m, and $|\psi\rangle = \sum_m c_m |m\rangle$, what is the probability that the system has energy E_m for some particular $m = n$? What is the *average* energy of the system?

2. Give the eigenstates and their energies for a spin 1/2 particle in an applied magnetic field, $\mathbf{B} = B\hat{z}$.

3. For a spin 1 particle the eigenstates of S_z are $\{|+\hat{z}\rangle, |0\hat{z}\rangle, |-\hat{z}\rangle\}$ with eigenvalues, respectively, $+\hbar$, 0, and $-\hbar$. However, the action of S_x is:

$$S_x|+\hat{z}\rangle = (\hbar/\sqrt{2})|0\hat{z}\rangle, \quad S_x|0\hat{z}\rangle = (\hbar/\sqrt{2})(|+\hat{z}\rangle + |-\hat{z}\rangle), \quad S_x|-\hat{z}\rangle = (\hbar/\sqrt{2})|0\hat{z}\rangle.$$

   (i) Represent S_z and S_x as 3×3 matrices in the $\hat{z}$ basis.

   (ii) Express the ket-vector $|+\hat{x}\rangle$ as a linear combination in the $\hat{z}$ basis.

   (iii) What is the inner product $\langle 0\hat{z}|+\hat{x}\rangle$?