Quantum Mechanics Review: homework #2

Recall that $|\uparrow\rangle$ and $|\downarrow\rangle$ correspond to spin-1/2 up or down, respectively, along $\hat{z}$ (*i.e.* $S_z = \pm\hbar/2$). Similarly $|\pm\hat{x}\rangle = (|\uparrow\rangle \pm |\downarrow\rangle)/\sqrt{2}$ correspond to states that have definite angular momentum $\pm\hbar/2$ along $\hat{x}$.

1. Review our Wednesday lecture, then rewrite the pure state density operator $\rho = |+\hat{x}\rangle\langle+\hat{x}|$ in terms of $|\uparrow\rangle$, $|\downarrow\rangle$, $\langle\uparrow|$, and $\langle\downarrow|$. You should have a total of four terms. One of these is $\frac{1}{2}|\uparrow\rangle\langle\uparrow|$. What are the other terms?

2. Use the preceding expression to write ρ as a matrix using the basis set $\{\uparrow,\downarrow\}$. Check that $\text{Tr}\,(\rho) = 1$ (necessary for a density matrix) and that $\rho = \rho^2$ (necessary for a pure state).

3. Express the mixed state density operator $\rho = |\uparrow\rangle P_1 \langle\uparrow| + |\downarrow\rangle P_1 \langle\downarrow|$ as a matrix using the basis set $\{\uparrow,\downarrow\}$. Check to be sure that your result is a diagonal matrix. Also check that $\text{Tr}\,(\rho) = 1$ (necessary for a density matrix) and that $\text{Tr}\,(\rho^2) < 1$ (necessary for a mixed state).

4. Using the same basis set $\{\uparrow,\downarrow\}$, express the Hamiltonian of a spin-1/2 particle in a magnetic field as a matrix. Recall that $H|\uparrow\rangle = \mu_B B|\uparrow\rangle$ and $H|\downarrow\rangle = -\mu_B B|\downarrow\rangle$. Apply the rule $\langle A\rangle = \text{Tr}\,(\rho A)$ to find the average energy, $\langle H\rangle$, in the mixed state defined previously.