Quantum Mechanics Review: homework #2

Recall that \(|\uparrow\rangle\) and \(|\downarrow\rangle\) correspond to spin-1/2 up or down, respectively, along \(\hat{z}\) (i.e. \(S_z = \pm \hbar/2\)). Similarly \(|\pm \hat{x}\rangle = (|\uparrow\rangle \pm |\downarrow\rangle)/\sqrt{2}\) correspond to states that have definite angular momentum \(\pm \hbar/2\) along \(\hat{x}\).

1. Review our Wednesday lecture, then rewrite the pure state density operator \(\rho = |+\hat{x}\rangle\langle+\hat{x}|\) in terms of \(|\uparrow\rangle\), \(|\downarrow\rangle\), \(\langle\uparrow|\), and \(\langle\downarrow|\). You should have a total of four terms. One of these is \(\frac{1}{2}|\uparrow\rangle\langle\uparrow|\). What are the other terms?

2. Use the preceding expression to write \(\rho\) as a matrix using the basis set \{\(\uparrow, \downarrow\}\}. Check that \(\text{Tr} (\rho) = 1\) (necessary for a density matrix) and that \(\rho = \rho^2\) (necessary for a pure state).

3. Express the mixed state density operator \(\rho = |\uparrow\rangle P_1 \langle\uparrow| + |\downarrow\rangle P_2 \langle\downarrow|\) as a matrix using the basis set \{\(\uparrow, \downarrow\}\}. Check to be sure that your result is a diagonal matrix. Also check that \(\text{Tr} (\rho) = 1\) (necessary for a density matrix) and that \(\text{Tr} (\rho^2) < 1\) (necessary for a mixed state).

4. Using the same basis set \{\(\uparrow, \downarrow\}\}, express the Hamiltonian of a spin-1/2 particle in a magnetic field as a matrix. Recall that \(H|\uparrow\rangle = \mu_B B|\uparrow\rangle\) and \(H|\downarrow\rangle = -\mu_B B|\downarrow\rangle\). Apply the rule \(\langle A\rangle = \text{Tr} (\rho A)\) to find the average energy, \(\langle H\rangle\), in the mixed state defined previously.