Quantum entropy from density matrix

Recall the density matrix is given by the exponential of the Hamiltonian,

$$\rho = \frac{1}{Z} e^{-\beta H}, \quad Z = \mathrm{Tr} e^{-\beta H}.$$

The *logarithm* of ρ is a matrix L with the property that $e^L = \rho$.

1. This problem considers the case of a spin-1/2 particle in a magnetic field. The energy levels are $\pm MB$. Express H and ρ matrices in the basis of energy eigenstates $\{|n\rangle\}$.

2. Determine the logarithm $L = \log \rho$.

3. Compute the entropy $S = -k_{\mathrm{B}} \, \mathrm{Tr}\{\rho \log \rho\}$ and graph it over the range $0 < k_{\mathrm{B}} T < 3MB$. Examine the limits $T \to 0$ and $T \to \infty$.