NAME: ____________________________________

33-448 Solid State Physics   Midterm #3   Monday, April 4, 2016

Note that problems 1 and 2 require brief written explanations of reasoning in a few sentences without lengthy derivation.

1. Heat capacity

The electronic density of states varies as $D(E) \sim E^{-1+d/2}$ in spatial dimension d, yet the electronic heat capacity varies as $C_{el} \sim T^p$, where the power p is independent of dimension d. Briefly explain the value of p. Why does it not depend on dimension, while the power law for vibrational heat capacity (C_{vib}) does depend on d?

2. Pauli paramagnetism

Every electron has magnetic moment $\pm\mu\hat{z}$. In the presence of a magnetic field $\mathbf{B} = B\hat{z}$, electrons with moment "up" (parallel to the magnetic field) drop in energy by $-\mu B$, while those with moment down (opposite to the field) rise in energy by $+\mu B$. Some electrons flip their moments to align with the applied field so as to reduce the total energy. This effect is illustrated in the figure below, taken from Kittel chapter 11. Let $T = 0$K (absolute zero) in the following.

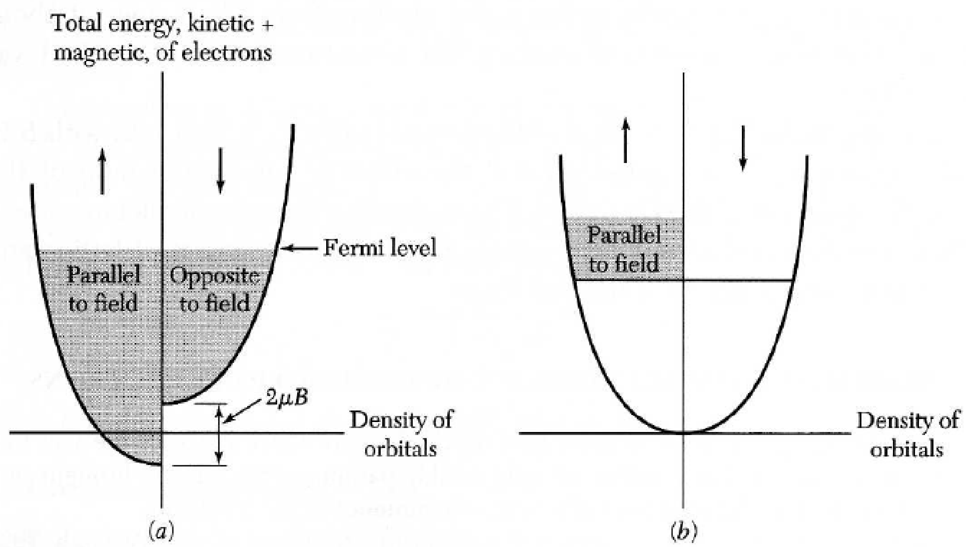

Figure 10 Pauli paramagnetism at absolute zero; the orbitals in the shaded regions in (a) are occupied. The numbers of electrons in the "up" and "down" band will adjust to make the energies equal at the Fermi level. The chemical potential (Fermi level) of the moment up electrons is equal to that of the moment down electrons. In (b) we show the excess of moment up electrons in the magnetic field.

Let N_+ be the numer of spin up electrons, N_- the number of spin down, and $M \equiv (N_+ - N_-)\mu$ be the net magnetization. For weak applied fields, the magnetization $M = [\mu]^p[D(E_F)]^q[B]^r$, where p, q and r are powers of moment, Fermi level density of states and field, respectively. Without any explicit derivation, state the values of p, q and r and briefly justify your assertions.

THIS PAGE LEFT BLANK

3. Frequency-dependent conductivity (adapted from Kittel #6.6)
Recall the drift velocity equation $m\,(d/dt + 1/\tau)\,v = qE$ for particles of mass m and charge q. Let the electric field oscillate as $E = \mathcal{E}_0 e^{-i\omega t}$ with $\omega > 0$ a fixed value.
(a) Derive a formula for the frequency-dependent complex conductivity $\sigma(\omega) \equiv j/E$, where j is the current density. Show the steps of your derivation, define any additional quantities you may need to introduce, and express your answer in the form $\sigma(\omega) = \sigma_0 f(\omega)$, where σ_0 is the static conductivity.

(b) Show that the current j falls out of phase with the electric field E in the limit of large relaxation time τ. What causes this phase lag, and what impact does it have on the average energy dissipation?