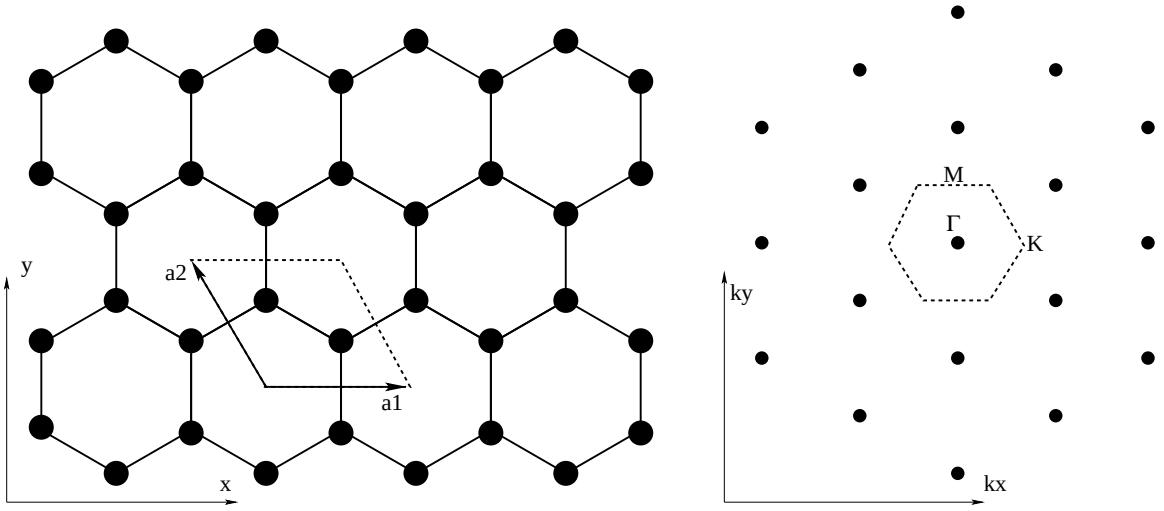
NAME: _____

33-448 Solid State Physics Final Exam #3 Tuesday, May 3, 2016

1. Band structure of graphene

The figure below illustrates the real-space structure and diffraction pattern of graphene. The dashed lines outline the unit cell in real space and the first Brillouin zone in reciprocal space. Special reciprocal points are marked Γ (center of zone), K (corner of zone) and M (mid-edge of zone). The reciprocal lattice vectors are

$$\mathbf{b}_1 = \frac{2\pi}{a}(\hat{x} + \frac{1}{\sqrt{3}})\hat{y} \qquad \mathbf{b}_2 = \frac{4\pi}{\sqrt{3}a}\hat{y}.$$



(a) Sketch and label the vectors $\pm\mathbf{b}_1$, $\pm\mathbf{b}_2$, and $\pm(\mathbf{b}_1 - \mathbf{b}_2)$ on the diffraction pattern.

(b) Let the electron-lattice interaction potential at $\mathbf{r} = (x, y)$ be

$$U(\mathbf{r}) = U_0 \left[\cos\left(\frac{2\pi}{a}(x + \frac{1}{\sqrt{3}}y) \right) + \cos\left(\frac{2\pi}{a}(x - \frac{1}{\sqrt{3}}y) \right) + \cos\left(\frac{4\pi}{\sqrt{3}a}y \right) \right].$$

Verify that $U(\mathbf{r})$ obeys the required translational symmetries of the crystal lattice. Hint: this is easiest if you think about reciprocal space, and I will give a simple answer more points than a complicated one.

(c) Based on your knowledge of the origin of band gaps, estimate the band gap E_g at the special point M predicted by the nearly free electron model. Be sure to fully define any symbols or quantities you introduce. You may wish to recall the "central equation",

$$(\lambda_{\mathbf{k}} - E)C(\mathbf{k}) + \sum_{\mathbf{G}} U_{\mathbf{G}} C(\mathbf{k} - \mathbf{G}) = 0,$$

but do **not** attempt to solve it.

(c) continued

(d) Is there a band gap at the special point K? Briefly explain why or why not.

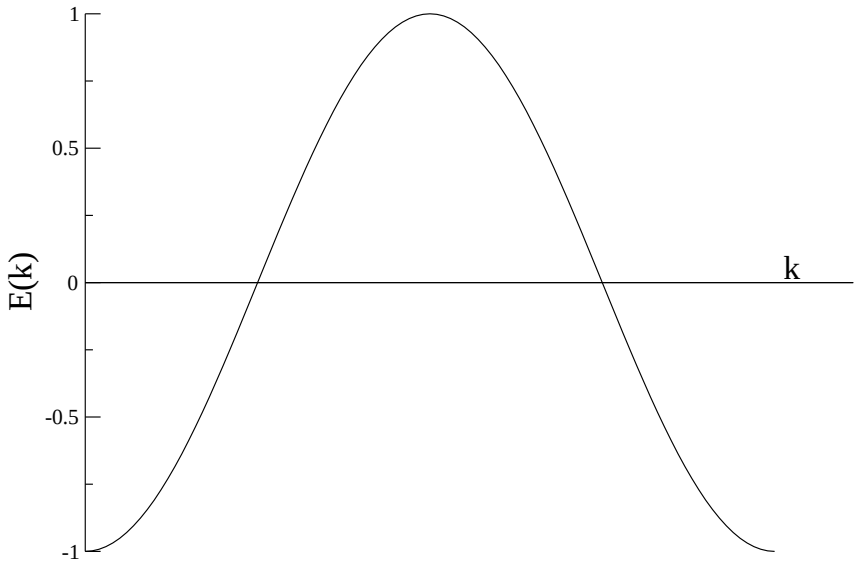
2. Bloch oscillations

Consider an electron moving in a one-dimensional periodic potential $V(x+a) = V(x)$. The electron accelerates in the presence of an applied force F, but as long as F is finite the energy must change continuously. Hence the electron energy is confined to a single band. Let the dispersion relation be $E(k) = -A\cos(bk)$ within this band.

(a) What is the value of b? What is the *group* velocity of the electron with wavenumber k?

(b) A constant applied electric field $\vec{\mathcal{E}} = -\mathcal{E}\hat{x}$ accelerates the electron. What is dk/dt? Neglect any scattering by impurities, defects or phonons, *i.e.* take the mean free time $\tau \to \infty$.

(c) Assume the electron is initially localized around the origin $x \approx 0$ in a wave packet with $k \approx 0$. Qualitatively describe the motion of the electron in response to the applied field, and comment on how the motion differs from that of a free particle. Use the figure below to assist your explanation.



3. Quantum well density of states

(a) For free electron motion in two dimensions, the electronic density of states $D(E) \sim E^p$. State the value of p and briefly justify your answer.

(b) A quantum well formed in a semiconductor heterostructure can be modeled with a potential

$$V(x,y,z) = \begin{cases} 0 & (|z| < a/2) \\ +\infty & (a/2 < |z|). \end{cases}$$

On the axes below, sketch the density of states $D(E)$ for this quantum well and comment on any notable features. On this figure, the units of energy are $[(\hbar^2/2m)(\pi/a)^2]$ and the units of $D(E)$ are arbitrary. You may wish to recall the solution of the "particle in a box" problem,
$E_n = (\hbar^2/2m)(n\pi/a)^2$ with $n \geq 1$ a positive integer.