

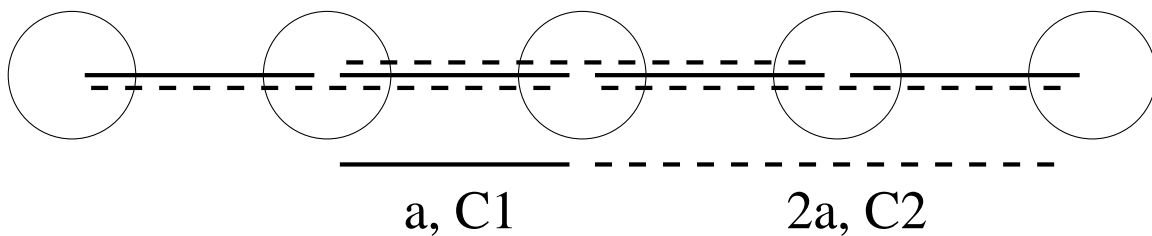
NAME: \_\_\_\_\_

33-448 Solid State Physics      Midterm #2      Monday, March 27, 2017

Please read each question carefully before answering. Do not do unnecessary work - it will waste time and might cost you points. Be sure to attempt every part, as some can be solved independently of others.

1. Dispersion relation for next-nearest neighbor chain (adapted from Simon #9.3)

A monatomic chain of atoms with mass  $m$  has nearest neighbor springs of unstretched length  $a$  and spring constant  $C_1$ , and second neighbor springs of unstretched length  $2a$  and spring constant  $C_2$ .



(a) Write down the equation of motion for the  $n^{\text{th}}$  atom in the chain.

(b) Determine the dispersion relation  $\omega(k)$ . Hint: the solution is almost trivial if you remember that the functional form of normal modes is governed by the symmetries of the system under consideration.

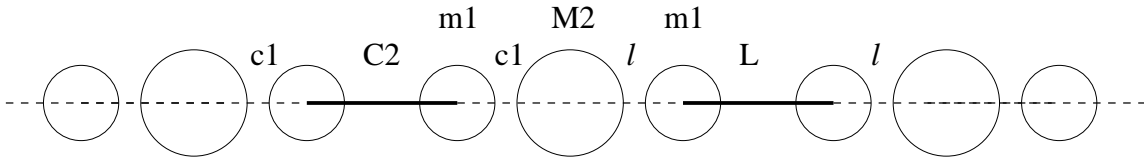
(c)

i. What is the sound speed  $v_s$ ?

ii. Discuss the relative proportions of  $C_1$  and  $C_2$  in your expression for  $v_s$ .

2. Triatomic chain

(a) Consider the triatomic chain illustrated below. The masses are  $m_1$  and  $M_2 > m_1$ , the bond lengths are  $\ell$  and  $L > \ell$ , and their spring constants are  $c_1$  and  $C_2 > c_1$ . Sketch the atomic displacements in the highest frequency mode using arrows below all the atoms. Derive (in the simplest way possible!) the frequency of this mode. What is its wavenumber  $k$ ?



(b) Use the sound speed  $v_s = a\sqrt{C/m}$  of a monatomic chain to write down (without calculation) the sound speed of this triatomic chain. Justify your answer in words, using physical insight instead of equations. Hint: How do spring constants add in series?