1. Faculty course evaluation Have you completed your FCE, or will you do so? Yes (1 point)/No (0 points)

2. Brillouin zones and Fermi surfaces

(a) Examine the body centered cubic (BCC) first Brillouin zone shown above, left. The special symmetry points H and N sit at $\mathbf{G}_H/2$ and $\mathbf{G}_N/2$, where $\mathbf{G}_H$ and $\mathbf{G}_N$ are reciprocal lattice vectors. State the Miller indices of $\mathbf{G}_H$ and $\mathbf{G}_N$ and briefly justify your assertion. Be sure to use the conventional (simple cubic) indexing, as is usual for BCC structures.
(b) Now consider the Fermi surface illustrated on the right. Is the material a metal, or an insulator? What is the valence? Briefly justify your conclusions.

(c) The H-point (sharp corner) sits a distance $1.91/\text{Å}^{-1}$ away from the Γ-point. By visual inspection of the figure, estimate the Fermi wavenumber $k_F$.

(d) In order to calculate the electron dispersion relation $E(k)$ of this material, which would be a more accurate starting point, a tight-binding model or the nearly-free electron approximation? Why?
3. Magnetoconductivity and Hall effect

(a) A $p$-type semiconductor with effective mass $m$ is placed in a magnetic field $\mathbf{B} = (0, 0, B_z)$ with $B_z > 0$. In which direction do the charge carriers rotate when viewed from above, clockwise or counterclockwise?

(b) The semiconductor is now placed in an electric field $\mathbf{E} = (E_x, E_y, 0)$. Determine the resulting steady-state current density $\mathbf{j}$. Please express your result in terms of the Drude conductivity $\sigma_0 = \frac{pe^2}{m} \tau$, the cyclotron frequency $\omega_c = \frac{eB_z}{m}$ and the relaxation time $\tau$. The matrix relating $\mathbf{j}$ to $\mathbf{E}$ is known as the magnetoconductivity tensor $\sigma$. You only need concern yourself with the $x$ and $y$ components of $\mathbf{j}$ and $\mathbf{E}$. 
(c) In the Hall geometry, a current $j_x$ is passed through the sample and an electric field $E_y$ results. What is the physical origin of $E_y$? Derive the value of $E_y$ as a function of $j_x$, $B$, and any other relevant quantities.
4. Peierls transition

An infinite one dimensional chain of monovalent atoms has bonds of length $a$ and spring constant $C$. The electronic band structure can be described by a tight-binding model with onsite energy $\epsilon_0 = 0$ and nearest neighbor electron hopping amplitude (matrix element) $-t$ (with $t > 0$).

\[ a \]

(a) Sketch the electron dispersion relation on the axes provided, and label the Brillouin zone boundaries, the Fermi energy $E_F$, and the Fermi wavevector $k_F$.

\[ E \]
\[ k \]
\[ -\pi/a \]
\[ +\pi/a \]

(b) Calculate the band energy $E_b$, defined as the energy per atom.
(c) The chain distorts causing alternating displacements of $\pm \delta$ as shown. What is the wavenumber $K$ of this distortion? What is the elastic energy per atom $E_d$ given the distortion amplitude $\delta$?

(d) Due to the displacement, the bond lengths alternate short and long, with hopping amplitudes $t_{S,L} = t \pm \delta$. Sketch the electron dispersion relation on the axes provided, and label the Brillouin zone boundaries, the Fermi energy $E_F$, and the Fermi wavevector $k_F$. Your sketch should be accurate, but need not be exact.
(e) Which energy change is greater in magnitude ($|E_b|$ or $|E_d|$) for small $\delta$? Will the chain spontaneously distort? Briefly justify your answer. Hint, the band gap is linear in $\delta$.

(f) Calculate the band gap $E_g$ exactly. Hint: this easiest if you can guess the eigenstates at the band edges.