Superconducting critical field

As discussed in Kittel (Appendix I), the Ginzburg-Landau free energy density of a superconducting material in the presence of an external magnetic field B_e is

$$f_S = f_N - \alpha|\psi|^2 + \frac{1}{2}\beta|\psi|^4 + \frac{1}{2m}|(\frac{\hbar}{i}\nabla - q\mathbf{A}/c)\psi|^2 - \int_0^{B_e} \mathbf{M}\cdot \mathrm{d}\mathbf{B}$$

(a) The order parameter $\psi = \sqrt{n}e^{i\theta}$ vanishes in the normal state, and becomes nonzero in the superconducting state provided $\alpha = T_c - T > 0$. Show that the pair density $n = \alpha/\beta$, and that $f_S - f_N = -\alpha^2/2\beta$, in the absence of a magnetic field.

(b) The fourth (final) term represents the magnetic work done to exclude the external field. Assuming perfect diamagnetism ($\chi = -1/4\pi$), determine the work done when the external field reaches the value B_e.

(c) The free energy of the normal state is not affected by the external field B_e. In the superconducting state, the presence of a vector potential, θ varies so as to cause the kinetic energy term (the third term above) to vanish. Determine the critical value of the external field B_e at which the free energy of the superconducting state f_S equals the free energy of the normal state f_N, for a type I superconductor.