Landau levels (adapted from Kittel 9.11)

The Hamiltonian for a spinless charged particle (charge q) in a magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$ with vector potential $\mathbf{A}$ is

$$H = \frac{1}{2m}\left(\mathbf{P} - q\mathbf{A}\right)^2.$$

In the Landau gauge $\mathbf{A} = -By\hat{\mathbf{x}}$. Assuming motion confined to the xy-plane, the Hamiltonian becomes

$$H = \frac{1}{2m}\left(-i\hbar\frac{\partial}{\partial x} + \frac{q}{c}yB\right)^2 - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial y^2}$$

(a) Because the Landau gauge breaks symmetry in the direction $\hat{\mathbf{y}}$ but preserves it in the direction $\hat{\mathbf{x}}$, the Schrodinger equation $H\psi = E\psi$ has solutions of the form $\psi(x,y) = e^{ikx}\chi(y)$. Show that $\chi(y)$ obeys an equation of the form $H_y\chi(y) = E_y\chi(y)$ with

$$H_y = \frac{\hbar^2}{2m}\frac{d^2}{dy^2} - \frac{1}{2}m\omega_c^2(y - y_0)^2.$$

(b) Show that the allowed values of E_y are quantized as $E_n = (n + 1/2)\hbar\omega_c$.

(c) Let the particle be confined to a region of size $L_x \times L_y$, and determine the degeneracy of the nth Landau level E_n.