

## Pauli paramagnetism (adapted from Kittel 11.5)

Let  $N_{\pm} = (1 \pm \zeta)N/2$  be the numbers of spin-up and -down electrons.

(a) Show that in a magnetic field  $B$  the total energy of the spin-up band is

$$E^+ = E_0(1 + \zeta)^{5/3} - \frac{1}{2}N\mu_B B(1 + \zeta),$$

where  $E_0 = 3NE_F/10$  and  $E_F$  is the zero-field Fermi energy. Find a similar expression for  $E^-$ .

(b) Expand  $E_{\text{total}} = E^+ + E^-$  to second order in  $\zeta$  around  $\zeta = 0$ .

(c) Determine the magnetization  $M$  to first order in  $B$ , and the magnetic susceptibility  $\chi$  in the limit  $B \rightarrow 0$ . Your results should be expressed in terms of the ratio  $N/E_F$ .