3.1 Drude theory of metals

a) Recall the relaxation time equation of motion
\[ \frac{d\vec{p}}{dt} = \vec{F} - \frac{\vec{p}}{\tau} \quad \text{with} \quad \vec{F} = -e\vec{E} \]

In steady state \( \frac{d\vec{p}}{dt} = 0 \Rightarrow \vec{p} = -e\vec{E} \)

**Current** \( \vec{j} = -ne\vec{v} = -ne\vec{p}/m = \left( \frac{ne^2\tau}{m} \right) \vec{E} \)

\( \sigma \) Conductivity

b) Now let \( \vec{F} = -e(\vec{E} + \vec{v} \times \vec{B}) \)

We take \( \vec{B} = B\hat{z} \)

**Steady state** \( \Rightarrow \vec{E} = -e\tau(e\vec{E} + \vec{v} \times \vec{B}) - \vec{p} \)

**Solve for** \( \vec{E} = \left( \frac{m}{ne^2\tau} \right) \vec{v} + \frac{1}{ne} \vec{p} \times \vec{B} = \vec{p} \cdot \vec{g} \)

Evaluating \( \vec{g} \times \vec{B} = j \times \vec{B} - j \times 0 \vec{y} \), we find

\[ \vec{p} = \left( \frac{m}{ne^2\tau} \right) \begin{pmatrix} 1 & \omega \tau & 0 \\ -\omega \tau & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

where \( \omega = eB/m \) is the cyclotron frequency.

\[ \vec{E} = \frac{ne^2\tau}{1 + (\omega \tau)^2} \begin{pmatrix} 1 & -\omega \tau & 0 \\ \omega \tau & 1 & 0 \\ 0 & 0 & 1 + (\omega \tau)^2 \end{pmatrix} \]

\( e^2 \approx \frac{ne^2\tau}{m} \frac{1}{1 + (\omega \tau)^2} \)

\( c) \) In a Hall geometry with current \( jx \) and field \( Bz \),

the Hall coefficient \( RH = E_y/j \times Bz \)
Square bar of Sodium (Na)

Cross-section L x L with L = 5 mm

\[ V_y = E_y L \quad I = j \times L^2 \quad R_H = \frac{1}{n_e} \text{ (assuming Drude model)} \]

Predict \( V_y = \frac{IB}{Lne} \)

Electron density \( n = \text{(valence = 1)} \times \text{(density } p = 1.9 \text{ g/cm}^3) \times N_A / (\text{mass } M = 23) \)

Avogadro's number \( N_A = 6.02 \times 10^{23} / \text{mole} \)

\[ n = 2.6 \times 10^{28} / \text{m}^3 \Rightarrow V_y = 5 \times 10^{-5} \text{ Volts} \]

Very small voltage! Could be drowned by environmental fluctuations. Try a lock-in amplifier - oscillate I at frequency \( \omega \) and detect response \( V_y \) at frequency \( \omega \).

d) Drude theory fails to explain \( R_H > 0 \) and non-integer \( neR_H \) with \( n \) the atomic density.

e) With AC field we are out of steady state -- \( dB/dt \) to

For simplicity we take \( \varepsilon \to \infty \).

Substitute \( p^2 = -nej^2 / m \) and \( \vec{\nu} = -nej \) into

\[ \frac{d\vec{B}}{dt} = -e \left( \vec{E} + \vec{\nu} \times \vec{B} \right) \Rightarrow \frac{d\vec{j}}{dt} = \frac{ne^2}{m} \vec{E} - \frac{e}{m} \vec{\nu} \times \vec{B} \]
Let \( \bar{B} = B \hat{z} \) and \( \bar{E} = E_x(t) \hat{x}, \omega_c = eB/m \)

Components of equation of motion: *(jz not interesting!)*

\[
\frac{dj_x}{dt} = \frac{ne^2}{m} E_x - \omega_c j_y
\]

\[
\frac{dj_y}{dt} = \omega_c j_x
\]

\[
\Rightarrow \frac{d^2j_x}{dt^2} = \frac{ne^2}{m} \frac{dE_x}{dt} - \omega_c^2 j_x
\]

Assume \( E_x(t) = E_x e^{i\omega t} \) and \( j_x(t) = j_x e^{i\omega t} \)

\[-\omega_c^2 j_x = \frac{ne^2}{m} (i\omega) E_x - \omega_c^2 j_x
\]

\[
j_x = 0 xx E_x \quad \text{with} \quad 0 xx = \frac{i\omega ne^2/m}{\omega_c^2 - \omega^2}
\]

\[
0 xy = \frac{\omega_e ne^2/m}{\omega_c^2 - \omega^2}
\]

\(0 xx\) diverges at \( \omega = \omega_c \equiv \text{cyclotron resonance}\).

Since \( \omega_c = eB/m \) and \( B \) is known, measuring \( \omega_c \) yields \( e/m \). In practice \( e \) is a universal constant but the mass of the electron can vary within a crystalline environment.
4.1 Free electron Fermi surface

a) The Fermi energy is the T=0 limit of chemical potential. In a metal this is the highest occupied state (or lowest unoccupied state). In a semiconductor with a gap the value lies within the gap. The Fermi temperature $T_F \equiv E_F/h\omega$ is a characteristic temperature for electrons. The Fermi surface is the surface in reciprocal space containing filled states.

b) Set the number of electrons $N$ to the reciprocal space density of states $\left(\frac{L}{2\pi}\right)^3$ multiplying the volume $\frac{4\pi}{3}k^3/2m$ due to spin (2) $k_F = (2\pi^2 N/V)^{1/3}$ ($V = L^3$), $E_F = \frac{\hbar^2 k^2}{2m}$

Note: $E_F \sim k_F^2 \sim N^{2/3}$

$\Rightarrow dE_F/dN = \left(\frac{2}{3}\right) E_F/N$

$\Rightarrow dN/dE_F = \left(\frac{3}{2}\right) N/E_F$

\begin{itemize}
  \item Electron density $N \equiv N/V = (V/\text{volume}) \times (\text{density } \rho = \text{total mass}/\text{volume})$ \times $N_A$ (atomic mass $23.334 g/mole$)
  \item From $E_F = \frac{\hbar^2 k^2}{2m} = 5 \times 10^{-19} J$ $= 2.6 \times 10^{-28} J/m^3$
\end{itemize}

We get $E_F = 3.2$ eV.