Thermodynamics of information

J.C. Maxwell Unified electricity and magnetism 1861

Introduced demon 1871

An intelligent being with knowledge of $\mathbf{r}$, $\mathbf{v}$

Ideal gas in thermal equilibrium

$$P = \rho k_B T$$

Maximum entropy $\Rightarrow \rho, T$ uniform

Select speed $\Rightarrow T$ imbalance $$\Rightarrow$$ violate 2nd law of thermodynamics?

Select direction $\Rightarrow p$ imbalance $$\Rightarrow$$ demon must get hot!

Leo Szilard (1929) Connect piston to do work:

1. Locate molecule
2. Remember position
3. Insert piston
4. Do work
5. Repeat

$$W = \int_{L/2}^{L} PdX$$

$$P = \rho k_B T$$

$$\rho = \sqrt{\frac{m}{2\pi k_B T}}$$

$$W = k_B T \int_{L/2}^{L} \frac{dx}{x}$$

$$= k_B T \ln 2$$

Isothermal conversion of heat to work violates 2nd law
Measurement must generate entropy $\Rightarrow$ in cyclic process

Leon Brillouin (IBM 1951) Information = (negative) entropy

Motion of piston destroys information of molecule position

Acquiring information raises entropy (e.g., light source to locate molecule)

2nd law: $S - I$ can never decrease
Rolf Landauer (IBM 1961) Computing is thermodynamic cyclic operation of demon requires information erasure.

Erasure creates entropy.

Computer logic is (usually) irreversible ⇒ destroys information.

E.g., AND gate throws away input bits.

\[ \begin{array}{c}
0 & 1 & 0 & 1 & A \\
0 & 0 & 1 & 1 & B
\end{array} \]

\[ \begin{array}{c}
0 & 0 & 0 & 1 \\
- & - & - & - & ?
\end{array} \]

Example: this iPad (10 Watts)

3 GHz 64 bit

\[ dQ = T dS \]

\[ = (3 \times 10^9 \, \text{sec}^{-1}) \\
\times 64 \times 1.4 \times 10^{-2} \, \text{J/K} \\
\times 300 \, \text{K} \times \ln 2 \]

\[ = 6 \times 10^{-9} \, \text{J/sec}. \]

\[ \uparrow \text{Watt} \]

Charles Bennett (IBM 1973) Reversible computing

Turing Machine - tape with program + blank space

\[ \text{moveable read/write head} \]

\[ \begin{array}{c}
0 & 1 & 1 & 0 & 1 & 0
\end{array} \]

Bennett adds a second tape with intermediate results. Running program backwards erases second tape. Physically reversible in slow limit.
Thermodynamic reversibility

Adiabatic expansion of gas

Isothermal expansion $dQ_T > 0$

$T$ work done $dW > 0$

Carnot cycle

Engine (clockwise) transfers heat from Hot to Cold doing work $dW > 0$.

Refrigerator (counterclockwise) transfers heat from Cold to Hot. Requires work as input.

Reversible if engine drives refrigerator in slow limit.
Examples from my research

Entropy of aluminum

\[ S_i = \frac{3}{2} - \ln \beta \lambda^3 \]

\[ I = \frac{1}{2} \rho k_B \int d^3 \mathbf{x} \, \text{grad} \ln \rho(\mathbf{r}) \cdot \text{grad} \ln \rho(\mathbf{r}) \]

I due to covariance of atomic vibrations