33-658 Quantum Computing and Quantum Information Homework 4

1. For a noninteracting composite system of independent subsystems A and B, where $H_{AB} = H_A + H_B$, prove that the time evolution operator factorizes,

$$T(t) = e^{-iH_{AB}t/\hbar} = e^{-iH_A t/\hbar} e^{-iH_B t/\hbar}.$$

Is this true even if $H_A = S_x$ and $H_B = S_y$? Explain.

2. Solve Shumacher & Westmoreland problem 6.6.

Consider the GHZ state $|\Psi\rangle = (1/\sqrt{2})(|000\rangle - |111\rangle)$.

(a) Suppose Alice makes a Z measurement on her qubit. Show that the qubits of Bob and Charlie are in a product state, regardless of the measurement outcome.

(b) Suppose Alice makes an X measurement on her qubit. Show that the qubits of Bob and Charlie are in an entangled state, regardless of the measurement outcome.

3. Solve Shumacher & Westmoreland problem 6.9.

(a) Show that measurements of the Y components of $|\Psi_-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$ yields opposite results.

(b) Does there exist an "anti-singlet" state of two qubits for which measurements of parallel spin components always yield identical results? If so, write down the state vector. If not, give a proof that no state, product or entangled, can do this.

4. Bell measurement experiment with quantum gates.

(a) Express the `H` and `CX` (C-NOT) gates as matrices of the appropriate dimension, and verify that $H^2 = 1$ and $CX^2 = 1$.

(b) Analyze the behavior of the circuit in the figure below. Determine the quantum state at each time t, for every combination of $x, y \in \{0, 1\}$. Whenever the state is a product state, express it in product form. How are the values of a and b related to x and y?

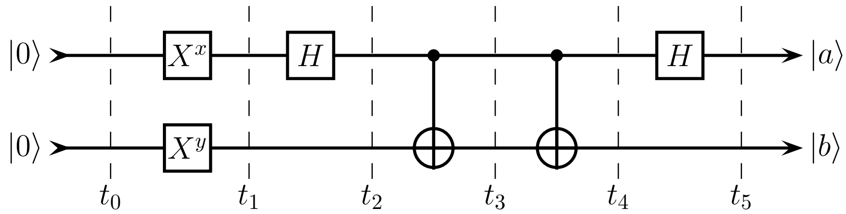

Figure 1: Bell state circuit.

(c) Program the case $x = 0$, $y = 1$ using IBM Composer and use the "Inspector" to check the quantum state at time t_3. Print a copy of a working circuit, then run it 1024 times on IBM-Q and print out the resulting histogram of (a, b) values.