33-658 Quantum Computing and Quantum Information Homework 5

1. Dense coding.

The Bell states defined by

$$|B_{00}\rangle = \big(|00\rangle + |11\rangle\big)/\sqrt{2}, \quad |B_{01}\rangle = \big(|01\rangle + |10\rangle\big)/\sqrt{2},$$
$$|B_{10}\rangle = \big(|00\rangle - |11\rangle\big)/\sqrt{2}, \quad |B_{11}\rangle = \big(|01\rangle - |10\rangle\big)/\sqrt{2}$$

form an orthonormal basis of fully entangled states on the tensor product $\mathcal{H}_a \otimes \mathcal{H}_b$ of two two-dimensional Hilbert spaces, where each space has an orthonormal basis $\{|0\rangle, |1\rangle\}$. Note that

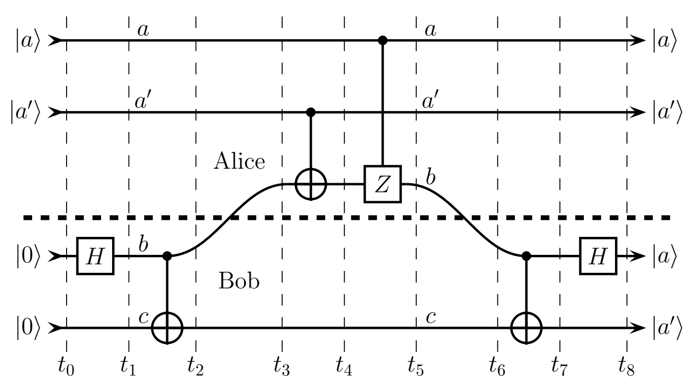
$$|B_{xy}\rangle = \frac{1}{\sqrt{2}}\left(|0y\rangle + (-1)^x |1\tilde{y}\rangle\right)$$

with $\tilde{y} = \neg y$.

(a) Find unitary operations of the form $U_a \otimes I_b$ that map $|B_{00}\rangle$ to $|B_{01}\rangle$, $|B_{10}\rangle$, and $|B_{11}\rangle$. Do you recognize the U_a operators?

(b) Alice and Bob share a Bell state B_{xy} but they do not know the values of x or y. Two bits of information are required to specify xy. How many bits of information concerning xy are obtained if Alice measures the value of her qubit a? How many bits of information are obtained if Bob also measures his qubit b and shares the result with Alice?

(c) Now let Alice and Bob each have two bits, $|a\rangle$ and $|a'\rangle$, and $|b\rangle$ and $|c\rangle$, respectively, as shown in the figure. Assume that $a, a' \in \{0,1\}$. The state at time 2 is a product state, $|\Psi_2\rangle = |a\rangle|a'\rangle|B_{00}\rangle$. Bob passes bit b to Alice, who acts on it between times 3 and 5, then returns it to Bob. What is the state at time 6?



(d) The figure claims that the final state at time 8 is $|aa'aa'\rangle$. Explain why this is true. Notice that two bits of classical information (a and a') have been shared by sharing a single qubit.

(e) The final state at time 8 contains two copies each of bits a and a'. Why does this not violate the no-cloning theorem?

2. Schumacher & Westmoreland problem #7.3 “Preparation machines”
3. Schumacher & Westmoreland problem #7.5 Teleportation.