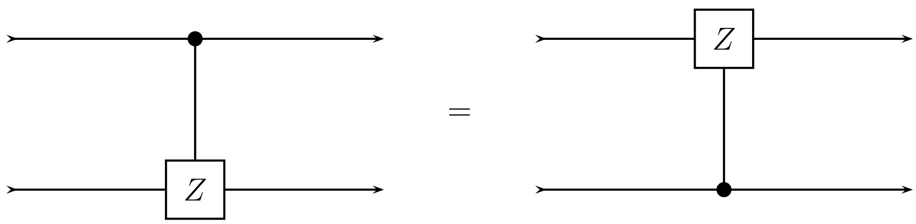
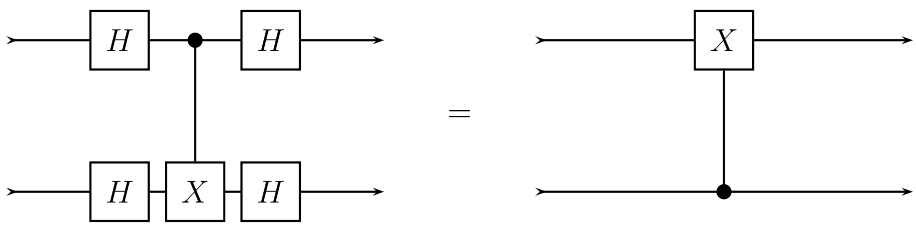
33-658 Quantum Computing and Quantum Information Homework 8

1. (i) prove $\mathbf{C}_{ij} = \tilde{\mathbf{n}}_i + \mathbf{n}_i\mathbf{X}_j$. (ii) Apply algebraic manipulations (i.e. do not use matrix arithmetic) to prove: $\mathbf{S}_{ij} = (1/2)(\mathbf{1} + \mathbf{X}_i\mathbf{X}_j + \mathbf{Y}_i\mathbf{Y}_j + \mathbf{Z}_i\mathbf{Z}_j) = (1/2)(1 + \vec{\sigma}_i \cdot \vec{\sigma}_j)$.

2. Prove the identities



and



3. In the Bernstein-Vazirani problem we are told that function $f(x) = a \cdot x$, where a and x are n-bit integers and the dot product

$$a \cdot x = a_0x_0 \oplus a_1x_1 \oplus \cdots \oplus a_{n-1}x_{n-1}$$

is the modulo 2 sum of products of bits. Suppose that we are given a quantum circuit that evaluates $f(x)$ for any input x but that we do not know the value of a. To determine a on a classical computer would require n evaluations of f. This exercise, which is based on the analysis in Mermin, shows how to determine a with a *single* evaluation of f on a quantum computer.
(i) Let $\mathbf{U}_f$ be an f-controlled unitary transformation on the tensor product of the n-bit input register initially in state $|x\rangle_n$ and a 1-bit output register initially in state $|y\rangle_1 = \mathbf{HX}|0\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$. Show that

$$\mathbf{U}_f|x\rangle_n|y\rangle_1 = (-1)^{f(x)}|x\rangle_n|y\rangle_1.$$

(ii) Show that

$$\mathbf{H}^{\otimes n}|x\rangle_n = \frac{1}{2^{n/2}}\sum_{y=0}^{2^n-1}(-1)^{x\cdot y}|y\rangle_n.$$

(iii) Show that

$$(\mathbf{H}^{\otimes n} \otimes \mathbf{H})\mathbf{U}_f(\mathbf{H}^{\otimes n} \otimes \mathbf{H})|0\rangle_n|1\rangle_1 = |a\rangle_n|1\rangle_1.$$

Note that the unknown value of a appears in the *input* register!